UNIT – II

Phase Controlled Converters- Single Phase

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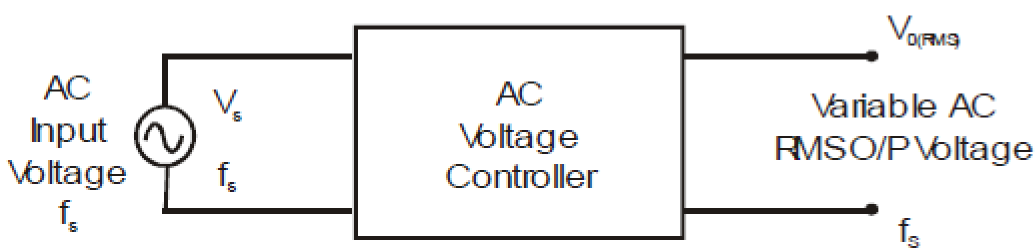
PHASE CONTROLLED CONVERTERS SINGLE PHASE (RMS VOLTAGE CONTROLLERS)

AC voltage controllers (ac line voltage controllers) are employed to vary the RMS value of the alternating voltage applied to a load circuit by introducing Thyristors between the load and a constant voltage ac source. The RMS value of alternating voltage applied to a load circuit is controlled by controlling the triggering angle of the Thyristors in the ac voltage controller circuits.

In brief, an ac voltage controller is a type of thyristor power converter which is used to convert a fixed voltage, fixed frequency ac input supply to obtain a variable voltage ac output. The RMS value of the ac output voltage and the ac power flow to the load is controlled by varying (adjusting) the trigger angle ‘α



There are two different types of thyristor control used in practice to control the acpower flow

- On-Off control
- Phase control

These are the two ac output voltage control techniques.

In On-Off control technique Thyristors are used as switches to connect the loadcircuit to the ac supply (source) for a few cycles of the input ac supply and then todisconnect it for few input cycles. The Thyristors thus act as a high speed contactor(or high speed ac switch).

PHASE CONTROL

In phase control the Thyristors are used as switches to connect the load circuitto the input ac supply, for a part of every input cycle. That is the ac supply voltage ischopped using Thyristors during a part of each input cycle.The thyristor switch is turned on for a part of every half cycle, so that inputsupply voltage appears across the load and then turned off during the remaining part of input half cycle to disconnect the ac supply from the load.By controlling the phase angle or the trigger angle ‘α’ (delay angle), theoutput RMS voltage across the load can be controlled.

The trigger delay angle ‘α’ is defined as the phase angle (the value of ωt) atwhich the thyristor turns on and the load current begins to flow.Thyristor ac voltage controllers use ac line commutation or ac phasecommutation. Thyristors in ac voltage controllers are line commutated (phasecommutated) since the input supply is ac. When the input ac voltage reverses andbecomes negative during the negative half cycle the current flowing through theconducting thyristor decreases and falls to zero. Thus the ON thyristor naturally turnsoff, when the device current falls to zero.Phase control Thyristors which are relatively inexpensive, converter grade

Thyristors which are slower than fast switching inverter grade Thyristors are normallyused.For applications upto 400Hz, if Triacs are available to meet the voltage andcurrent ratings of a particular application, Triacs are more commonly used.Due to ac line commutation or natural commutation, there is no need of extracommutation circuitry or components and the circuits for ac voltage controllers arevery simple.

Due to the nature of the output waveforms, the analysis, derivations ofexpressions for performance parameters are not simple, especially for the phasecontrolled ac voltage controllers with RL load. But however most of the practicalloads are of the RL type and hence RL load should be considered in the analysis anddesign of ac voltage controller circuits.

TYPE OF AC VOLTAGE CONTROLLERS

The ac voltage controllers are classified into two types based on the type ofinput ac supply applied to the circuit.

- Single Phase AC Controllers.
- Three Phase AC Controllers.

Single phase ac controllers operate with single phase ac supply voltage of230V RMS at 50Hz in our country. Three phase ac controllers operate with 3 phase ac Supply of 400V RMS at 50Hz supply frequency.

Each type of controller may be sub divided into

- Uni-directional or half wave ac controller.
- Bi-directional or full wave ac controller.

In brief different types of ac voltage controllers are

• Single phase half wave ac voltage controller (uni-directionalcontroller).

• Single phase full wave ac voltage controller (bi-directional controller).

• Three phase half wave ac voltage controller (uni-directionalcontroller).

• Three phase full wave ac voltage controller (bi-directional controller).

APPLICATIONS OF AC VOLTAGE CONTROLLERS

• Lighting / Illumination control in ac power circuits.

• Induction heating.

• Industrial heating & Domestic heating.

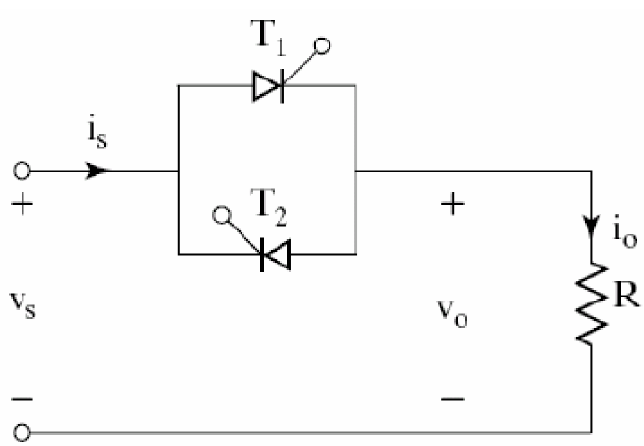
• Transformer tap changing (on load transformer tap changing).

• Speed control of induction motors (single phase and poly phase ac induction
motor control).

• AC magnet controls.

PRINCIPLE OF ON-OFF CONTROL TECHNIQUE (INTEGRAL CYCLECONTROL)

The basic principle of on-off control technique is explained with reference to asingle phase full wave ac voltage controller circuit shown below. The thyristorswitches T1 and T2 are turned on by applying appropriate gate trigger pulses toconnect the input ac supply to the load for 'n' number of input cycles during the timeinterval T_{ON} . The thyristor switches T1 and T2 are turned off by blocking the gatetrigger pulses for 'm' number of input cycles during the time interval T_{OFF} . The accontroller ON time t_{ON} usually consists of an integral number of input cycles.



$R = R_L =$ **Load Resistance**

Fig.1: Single phase full wave AC voltage controller circuit

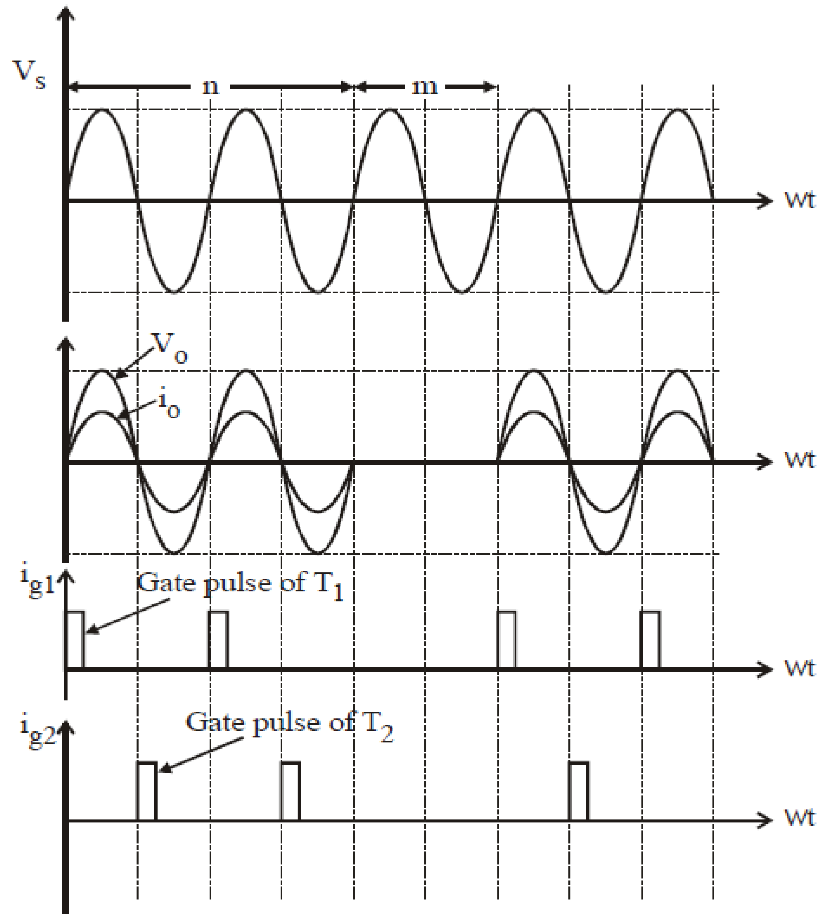


Fig.2: Waveforms

Example

Referring to the waveforms of ON-OFF control technique in the above diagram,

n = Two input cycles. Thyristors are turned ON during *ON t* for two inputcycles.

m = One input cycle. Thyristors are turned OFF during *OFF t* for one inputcycle

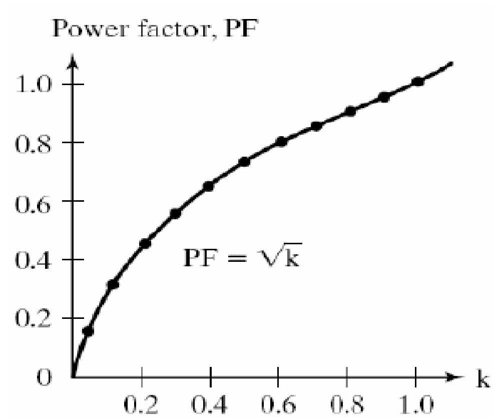


Fig.3: Power Factor

Thyristors are turned ON precisely at the zero voltage crossings of the inputsupply. The thyristor T1is turned on at the beginning of each positive half cycle byapplying the gate trigger pulses to T1 as shown, during the ON time *ON t* . The loadcurrent flows in the positive direction, which is the downward direction as shown inthe circuit diagram when T1 conducts. The thyristor T2 is turned on at the beginningof each negative half cycle, by applying gating signal to the gate of T2 , during T*ON* .The load current flows in the reverse direction, which is the upward direction whenT2 conducts. Thus we obtain a bi-directional load current flow (alternating loadcurrent flow) in a ac voltage controller circuit, by triggering the thyristors alternately.This type of control is used in applications which have high mechanical inertiaand high thermal time constant (Industrial heating and speed control of ac motors).Due to zero voltage and zero current switching of Thyristors, the harmonics generatedby switching actions are reduced.

For a sine wave input supply voltage,

$$v_s = V_m \sin \omega t = \sqrt{2} V_S \sin \omega t$$

V_S = RMS value of input ac supply = $\frac{V_m}{\sqrt{2}}$ = RMS phase supply voltage.

If the input ac supply is connected to load for 'n' number of input cycles and disconnected for 'm' number of input cycles, then

$$t_{ON} = n \times T, \qquad t_{OFF} = m \times T$$

Where $T = \frac{1}{f}$ = input cycle time (time period) and

f = input supply frequency.

t_{ON} = controller on time = $n \times T$.

t_{OFF} = controller off time = $m \times T$.

T_O = Output time period = $(t_{ON} + t_{OFF}) = (nT + mT)$.

We can show that,

Output RMS voltage $V_{O(RMS)} = V_{i(RMS)}\sqrt{\frac{t_{ON}}{T_O}} = V_S\sqrt{\frac{t_{ON}}{T_O}}$

Where $V_{i(RMS)}$ is the RMS input supply voltage $= V_S$.

TO DERIVE AN EXPRESSION FOR THE RMS VALUE OF OUTPUTVOLTAGE, FOR ON-OFF CONTROL METHOD.

Output RMS voltage $V_{O(RMS)} = \sqrt{\frac{1}{\omega T_O}\int_{\omega t=0}^{\omega t_{ON}} V_m^2 Sin^2\omega t.d\left(\omega t\right)}$

$$V_{O(RMS)} = \sqrt{\frac{V_m^2}{\omega T_O}\int_{0}^{\omega t_{ON}} Sin^2\omega t.d\left(\omega t\right)}$$

Substituting for $Sin^2\theta = \frac{1-Cos2\theta}{2}$

$$V_{O(RMS)} = \sqrt{\frac{V_m^2}{\omega T_O}\int_{0}^{\omega t_{ON}}\left[\frac{1-Cos2\omega t}{2}\right]d\left(\omega t\right)}$$

$$V_{O(RMS)} = \sqrt{\frac{V_m^2}{2\omega T_O}\left[\int_{0}^{\omega t_{ON}} d\left(\omega t\right) - \int_{0}^{\omega t_{ON}} Cos2\omega t.d\left(\omega t\right)\right]}$$

$$V_{O(RMS)} = \sqrt{\frac{V_m^2}{2\omega T_O}\left[\left(\omega t\right)\Big/_{0}^{\omega t_{ON}} - \frac{Sin2\omega t}{2}\Big/_{0}^{\omega t_{ON}}\right]}$$

$$V_{O(RMS)} = \sqrt{\frac{V_m^2}{2\omega T_O}\left[\left(\omega t_{ON} - 0\right) - \frac{\sin 2\omega t_{ON} - \sin 0}{2}\right]}$$

Now *ON t* = An integral number of input cycles; Hence

$$t_{ON} = T, 2T, 3T, 4T, 5T, ... \;\&\; \omega t_{ON} = 2\pi, 4\pi, 6\pi, 8\pi, 10\pi, ...$$

Where T is the input supply time period (T = input cycle time period). Thus we note that $\sin 2\omega t_{ON} = 0$

$$V_{O(RMS)} = \sqrt{\frac{V_m^2 \cancel{\omega} t_{ON}}{2 \cancel{\omega} T_O}} = \frac{V_m}{\sqrt{2}} \sqrt{\frac{t_{ON}}{T_O}}$$

$$V_{O(RMS)} = V_{i(RMS)} \sqrt{\frac{t_{ON}}{T_O}} = V_S \sqrt{\frac{t_{ON}}{T_O}}$$

Where $V_{i(RMS)} = \frac{V_m}{\sqrt{2}} = V_S =$ RMS value of input supply voltage;

$$\frac{t_{ON}}{T_O} = \frac{t_{ON}}{t_{ON} + t_{OFF}} = \frac{nT}{nT + mT} = \frac{n}{(n+m)} = k = \text{duty cycle (d)}.$$

$$V_{O(RMS)} = V_S \sqrt{\frac{n}{(m+n)}} = V_S \sqrt{k}$$

PERFORMANCE PARAMETERS OF AC VOLTAGE CONTROLLERS

• **RMS Output (Load) Voltage**

$$V_{O(RMS)} = \left[\frac{n}{2\pi(n+m)} \int_0^{2\pi} V_m^2 \sin^2 \omega t . d(\omega t) \right]^{1/2}$$

$$V_{O(RMS)} = \frac{V_m}{\sqrt{2}} \sqrt{\frac{n}{(m+n)}} = V_{i(RMS)} \sqrt{k} = V_S \sqrt{k}$$

$$V_{O(RMS)} = V_{i(RMS)} \sqrt{k} = V_S \sqrt{k}$$

Where $V_S = V_{i(RMS)} =$ RMS value of input supply voltag

• **Duty Cycle**

$$k = \frac{t_{ON}}{T_O} = \frac{t_{ON}}{(t_{ON} + t_{OFF})} = \frac{nT}{(m+n)T}$$

Where, $k = \frac{n}{(m+n)}$ = duty cycle (d).

• **RMS Load Current**

$$I_{O(RMS)} = \frac{V_{O(RMS)}}{Z} = \frac{V_{O(RMS)}}{R_L};$$ for a resistive load $Z = R_L$.

• **Output AC (Load) Power**

$$P_O = I^2_{O(RMS)} \times R_L$$

• **Input Power Factor**

$$PF = \frac{P_O}{VA} = \frac{\text{output load power}}{\text{input supply volt amperes}} = \frac{P_O}{V_S I_S}$$

$$PF = \frac{I^2_{O(RMS)} \times R_L}{V_{i(RMS)} \times I_{in(RMS)}};$$ $I_S = I_{in(RMS)}$ = RMS input supply current.

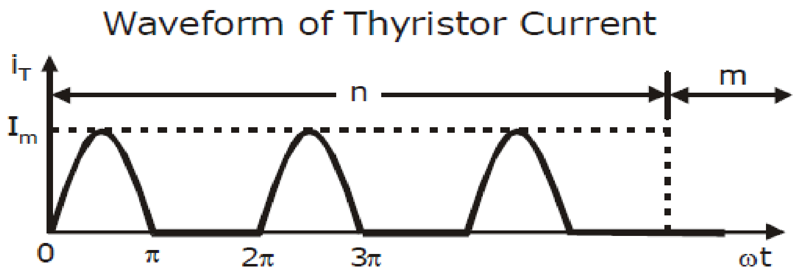
The input supply current is same as the load current $I_{in} = I_O = I_L$

Hence, RMS supply current = RMS load current; $I_{in(RMS)} = I_{O(RMS)}$.

$$PF = \frac{I^2_{O(RMS)} \times R_L}{V_{i(RMS)} \times I_{in(RMS)}} = \frac{V_{O(RMS)}}{V_{i(RMS)}} = \frac{V_{i(RMS)}\sqrt{k}}{V_{i(RMS)}} = \sqrt{k}$$

$$PF = \sqrt{k} = \sqrt{\frac{n}{m+n}}$$

• **The Average Current of Thyristor I** $T\ (Avg)$



$$I_{T(Avg)} = \frac{n}{2\pi(m+n)} \int_0^{\pi} I_m \sin\omega t.d(\omega t)$$

$$I_{T(Avg)} = \frac{nI_m}{2\pi(m+n)} \int_0^{\pi} \sin\omega t.d(\omega t)$$

$$I_{T(Avg)} = \frac{nI_m}{2\pi(m+n)} \left[-\cos\omega t \Big/_0^{\pi} \right]$$

$$I_{T(Avg)} = \frac{nI_m}{2\pi(m+n)} [-\cos\pi + \cos 0]$$

$$I_{T(Avg)} = \frac{nI_m}{2\pi(m+n)} [-(-1)+1]$$

$$I_{T(Avg)} = \frac{n}{2\pi(m+n)} [2I_m]$$

$$I_{T(Avg)} = \frac{I_m n}{\pi(m+n)} = \frac{k.I_m}{\pi}$$

$$k = \text{duty cycle} = \frac{t_{ON}}{(t_{ON} + t_{OFF})} = \frac{n}{(n+m)}$$

$$I_{T(Avg)} = \frac{I_m n}{\pi(m+n)} = \frac{k.I_m}{\pi},$$

Where $I_m = \frac{V_m}{R_L}$ = maximum or peak thyristor current.

• **RMS Current of Thyristor I** $T(RMS)$

$$I_{T(RMS)} = \left[\frac{n}{2\pi(n+m)}\int_0^{\pi} I_m^2 \sin^2 \omega t.d(\omega t)\right]^{\frac{1}{2}}$$

$$I_{T(RMS)} = \left[\frac{nI_m^2}{2\pi(n+m)}\int_0^{\pi} \sin^2 \omega t.d(\omega t)\right]^{\frac{1}{2}}$$

$$I_{T(RMS)} = \left[\frac{nI_m^2}{2\pi(n+m)}\int_0^{\pi} \frac{(1-\cos 2\omega t)}{2} d(\omega t)\right]^{\frac{1}{2}}$$

$$I_{T(RMS)} = \left[\frac{nI_m^2}{4\pi(n+m)}\left\{\int_0^{\pi} d(\omega t) - \int_0^{\pi} \cos 2\omega t.d(\omega t)\right\}\right]^{\frac{1}{2}}$$

$$I_{T(RMS)} = \left[\frac{nI_m^2}{4\pi(n+m)}\left\{(\omega t)\Big/_0^{\pi} - \left(\frac{\sin 2\omega t}{2}\right)\Big/_0^{\pi}\right\}\right]^{\frac{1}{2}}$$

$$I_{T(RMS)} = \left[\frac{nI_m^2}{4\pi(n+m)}\left\{(\pi - 0) - \left(\frac{\sin 2\pi - \sin 0}{2}\right)\right\}\right]^{\frac{1}{2}}$$

$$I_{T(RMS)} = \left[\frac{nI_m^2}{4\pi(n+m)}\{\pi - 0 - 0\}\right]^{\frac{1}{2}}$$

$$I_{T(RMS)} = \left[\frac{nI_m^2\pi}{4\pi(n+m)}\right]^{\frac{1}{2}} = \left[\frac{nI_m^2}{4(n+m)}\right]^{\frac{1}{2}}$$

$$I_{T(RMS)} = \frac{I_m}{2}\sqrt{\frac{n}{(m+n)}} = \frac{I_m}{2}\sqrt{k}$$

$$I_{T(RMS)} = \frac{I_m}{2}\sqrt{k}$$

PROBLEM

1. A single phase full wave ac voltage controller working on ON-OFF control technique has supply voltage of 230V, RMS 50Hz, load = 50Ω. The controller is ON for 30 cycles and off for 40 cycles. Calculate

- *ON & OFF time intervals.*
- *RMS output voltage.*

- *Input P.F.*
- *Average and RMS thyristor currents.*

$$V_{in(RMS)} = 230V\,, \qquad V_m = \sqrt{2} \times 230V = 325.269\,\text{V}, \quad V_m = 325.269V\,,$$

$$T = \frac{1}{f} = \frac{1}{50Hz} = 0.02\sec\,, \qquad T = 20ms\,.$$

n = number of input cycles during which controller is ON; $n = 30$.

m = number of input cycles during which controller is OFF; $m = 40$.

$$t_{ON} = n \times T = 30 \times 20ms = 600ms = 0.6\sec$$

$$t_{ON} = n \times T = 0.6\sec = \text{controller ON time.}$$

$$t_{OFF} = m \times T = 40 \times 20ms = 800ms = 0.8\sec$$
$$t_{OFF} = m \times T = 0.8\sec = \text{controller OFF time.}$$

Duty cycle $k = \dfrac{n}{(m+n)} = \dfrac{30}{(40+30)} = 0.4285$

RMS output voltage

$$V_{O(RMS)} = V_{i(RMS)} \times \sqrt{\frac{n}{(m+n)}}$$

$$V_{O(RMS)} = 230V \times \sqrt{\frac{30}{(30+40)}} = 230\sqrt{\frac{3}{7}}$$

$$V_{O(RMS)} = 230V\sqrt{0.42857} = 230 \times 0.65465$$

$$V_{O(RMS)} = 150.570V$$

$$I_{O(RMS)} = \frac{V_{O(RMS)}}{Z} = \frac{V_{O(RMS)}}{R_L} = \frac{150.570V}{50\Omega} = 3.0114A$$

$$P_O = I^2_{O(RMS)} \times R_L = 3.0114^2 \times 50 = 453.426498W$$

$$Input\ Power\ Factor\ \ P.F = \sqrt{k}$$

$$PF = \sqrt{\frac{n}{(m+n)}} = \sqrt{\frac{30}{70}} = \sqrt{0.4285}$$

$$PF = 0.654653$$

Average Thyristor Current Rating

$$I_{T(Avg)} = \frac{I_m}{\pi} \times \left(\frac{n}{m+n}\right) = \frac{k \times I_m}{\pi}$$

where $$I_m = \frac{V_m}{R_L} = \frac{\sqrt{2} \times 230}{50} = \frac{325.269}{50}$$

$$I_m = 6.505382A \text{ = Peak (maximum) thyristor current.}$$

$$I_{T(Avg)} = \frac{6.505382}{\pi} \times \left(\frac{3}{7}\right)$$

$$I_{T(Avg)} = 0.88745A$$

RMS Current Rating of Thyristor

$$I_{T(RMS)} = \frac{I_m}{2}\sqrt{\frac{n}{(m+n)}} = \frac{I_m}{2}\sqrt{k} = \frac{6.505382}{2} \times \sqrt{\frac{3}{7}}$$

$$I_{T(RMS)} = 2.129386A$$

PRINCIPLE OF AC PHASE CONTROL

The basic principle of ac phase control technique is explained with referenceto a single phase half wave ac voltage controller (unidirectional controller) circuitshown in the below figure.The half wave ac controller uses one thyristor and one diode connected inparallel across each other in opposite direction that is anode of thyristor T1 isconnected to the cathode of diode 1 *D* and the cathode of T1 is connected to the anodeof D1 . The output voltage across the load resistor ‘R’ and hence the ac power flow tothe load is controlled by varying the trigger angle ‘α’.The trigger angle or the delay angle ‘α’ refers to the value of ω*t* or the instantat which the thyristor T1is triggered to turn it ON, by applying a suitable gate triggerpulse between the gate and cathode lead.The thyristor T1 is forward biased during the positive half cycle of input acsupply. It can be triggered and made to conduct by applying a suitable gate triggerpulse only during the positive half cycle of input supply. When T1 is triggered itconducts and the load current flows through the thyristor T1 , the load and through thetransformer secondary winding.

By assuming T1 as an ideal thyristor switch it can be considered as a closedswitch when it is ON during the period $\omega t = \alpha$ to π radians. The output voltageacross the load follows the input supply voltage when the thyristor T1 is turned-on andwhen it conducts from $\omega t = \alpha$ to π radians. When the input supply voltage decreasesto zero at $\omega t = \pi$, for a resistive load the load current also falls to zero at $\omega t = \pi$ andhence the thyristor T1 turns off at $\omega t = \pi$. Between the time period $\omega t = \pi$ to 2π ,when the supply voltage reverses and becomes negative the diode D1 becomesforward biased and hence turns ON and conducts. The load current flows in theopposite direction during $\omega t = \pi$ to 2π radians when D1 is ON and the output voltagefollows the negative half cycle of input supply.

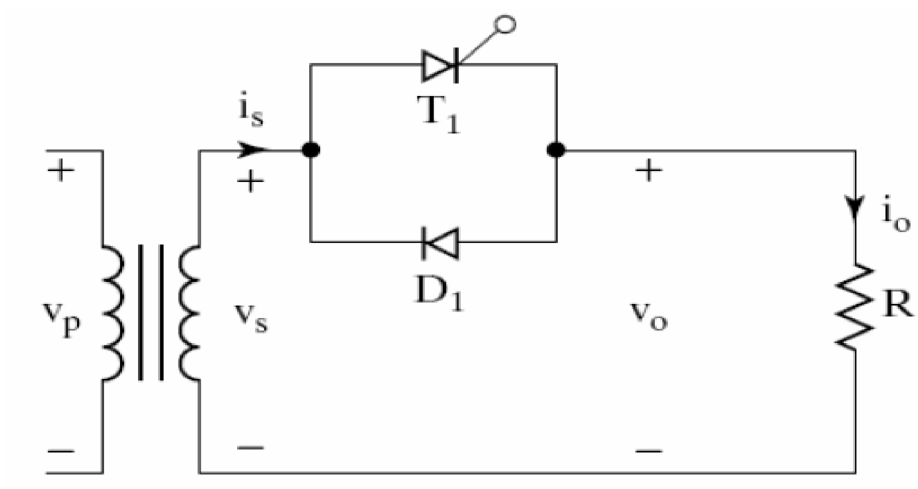
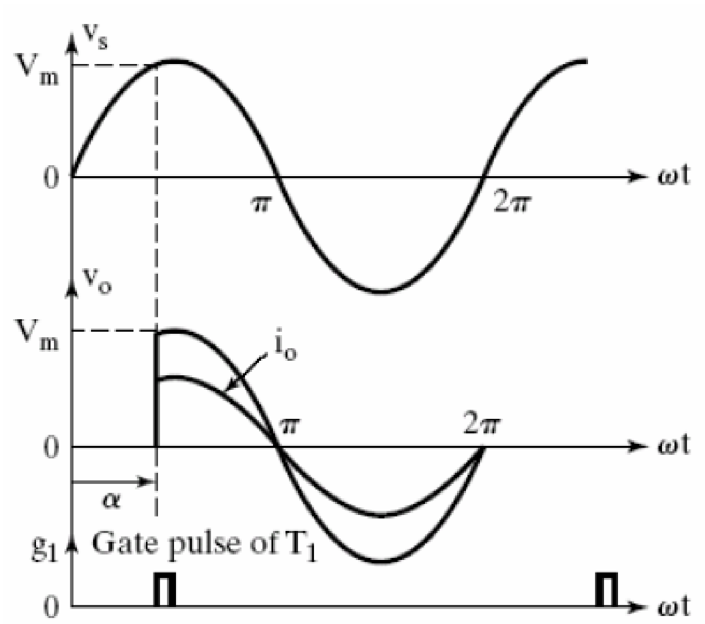


Fig.4: Halfwave AC phase controller (Unidirectional Controller)



Equations

Input AC Supply Voltage across the Transformer Secondary Winding.

$$v_s = V_m \sin \omega t$$

$$V_S = V_{in(RMS)} = \frac{V_m}{\sqrt{2}} = \text{RMS value of secondary supply voltage.}$$

Output Load Voltage

$$v_o = v_L = 0 \text{ ; for } \omega t = 0 \text{ to } \alpha$$

$$v_o = v_L = V_m \sin \omega t \text{ ; for } \omega t = \alpha \text{ to } 2\pi \text{ .}$$

Output Load Current

$$i_o = i_L = \frac{v_o}{R_L} = \frac{V_m \sin \omega t}{R_L} \text{ ; for } \omega t = \alpha \text{ to } 2\pi \text{ .}$$

$$i_o = i_L = 0 \text{ ; for } \omega t = 0 \text{ to } \alpha \text{ .}$$

TO DERIVE AN EXPRESSION FOR RMS OUTPUT VOLTAGE $V_{O(RMS)}$

$$V_{O(RMS)} = \sqrt{\frac{1}{2\pi}\left[\int_{\alpha}^{2\pi} V_m^{\ 2}\sin^2 \omega t.d\left(\omega t\right)\right]}$$

$$V_{O(RMS)} = \sqrt{\frac{V_m^{\ 2}}{2\pi}\left[\int_{\alpha}^{2\pi}\left(\frac{1-\cos 2\omega t}{2}\right).d\left(\omega t\right)\right]}$$

$$V_{O(RMS)} = \sqrt{\frac{V_m^{\ 2}}{4\pi}\left[\int_{\alpha}^{2\pi}\left(1-\cos 2\omega t\right).d\left(\omega t\right)\right]}$$

$$V_{O(RMS)} = \frac{V_m}{2\sqrt{\pi}}\sqrt{\left[\int_{\alpha}^{2\pi} d\left(\omega t\right) - \int_{\alpha}^{2\pi}\cos 2\omega t.d\omega t\right]}$$

$$V_{O(RMS)} = \frac{V_m}{2\sqrt{\pi}}\sqrt{\left[\left(\omega t\right)\Big/_{\alpha}^{2\pi} - \left(\frac{\sin 2\omega t}{2}\right)\Big/_{\alpha}^{2\pi}\right]}$$

$$V_{O(RMS)} = \frac{V_m}{2\sqrt{\pi}}\sqrt{\left(2\pi-\alpha\right) - \left(\frac{\sin 2\omega t}{2}\right)\Big/_{\alpha}^{2\pi}}$$

$$V_{O(RMS)} = \frac{V_m}{2\sqrt{\pi}}\sqrt{\left(2\pi-\alpha\right) - \left\{\frac{\sin 4\pi}{2} - \frac{\sin 2\alpha}{2}\right\}} \qquad ;\sin 4\pi = 0$$

$$V_{O(RMS)} = \frac{V_m}{2\sqrt{\pi}}\sqrt{\left(2\pi-\alpha\right) + \frac{\sin 2\alpha}{2}}$$

$$V_{O(RMS)} = \frac{V_m}{\sqrt{2}\sqrt{2\pi}}\sqrt{\left(2\pi-\alpha\right) + \frac{\sin 2\alpha}{2}}$$

$$V_{O(RMS)} = \frac{V_m}{\sqrt{2}}\sqrt{\frac{1}{2\pi}\left[\left(2\pi-\alpha\right) + \frac{\sin 2\alpha}{2}\right]}$$

$$V_{O(RMS)} = V_{i(RMS)} \sqrt{\frac{1}{2\pi}\left[(2\pi - \alpha) + \frac{\sin 2\alpha}{2}\right]}$$

$$V_{O(RMS)} = V_S \sqrt{\frac{1}{2\pi}\left[(2\pi - \alpha) + \frac{\sin 2\alpha}{2}\right]}$$

Where, $V_{i(RMS)} = V_S = \frac{V_m}{\sqrt{2}}$ = RMS value of input supply voltage (across the transformer secondary winding).

Note: Output RMS voltage across the load is controlled by changing 'α ' as indicatedby the expression for $V_{O(RMS)}$

PLOT OF $V_{O(RMS)}$ VERSUS TRIGGER ANGLE α FOR A SINGLE PHASEHALF-WAVE AC VOLTAGE CONTROLLER (UNIDIRECTIONALCONTROLLER)

$$V_{O(RMS)} = \frac{V_m}{\sqrt{2}} \sqrt{\frac{1}{2\pi}\left[(2\pi - \alpha) + \frac{\sin 2\alpha}{2}\right]}$$

$$V_{O(RMS)} = V_S \sqrt{\frac{1}{2\pi}\left[(2\pi - \alpha) + \frac{\sin 2\alpha}{2}\right]}$$

By using the expression for $V_{O(RMS)}$ we can obtain the control characteristics,which is the plot of RMS output voltage $V_{O(RMS)}$ versus the trigger angle α . A typicalcontrol characteristic of single phase half-wave phase controlled ac voltage controlleris as shown below

Trigger angle α in degrees	Trigger angle α in radians		$V_{O(RMS)}$
0	0		$V_S = \frac{V_m}{\sqrt{2}}$
30^0	$\pi/6$	; $\left(1\pi/6\right)$	0.992765 V_S
60^0	$\pi/3$	; $\left(2\pi/6\right)$	0.949868 V_S
90^0	$\pi/2$	; $\left(3\pi/6\right)$	0.866025 V_S
120^0	$2\pi/3$	; $\left(4\pi/6\right)$	0.77314 V_S
150^0	$5\pi/6$	; $\left(5\pi/6\right)$	0.717228 V_S
180^0	π	; $\left(6\pi/6\right)$	0.707106 V_S

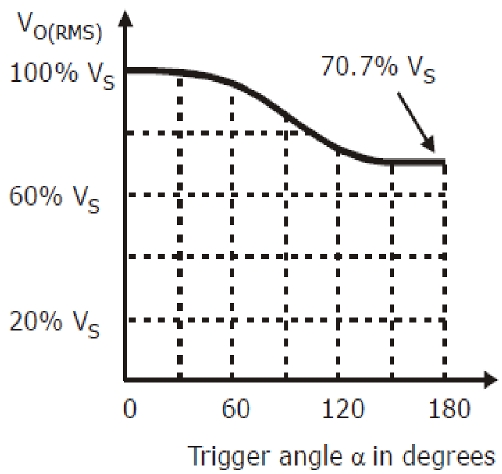


Fig.5: Control characteristics of single phase half-wave phase controlled acvoltage controller

Note: We can observe from the control characteristics and the table given above thatthe range of RMS output voltage control is from 100% of *S V* to 70.7% of *S V* when wevary the trigger angle α from zero to 180 degrees. Thus the half wave ac controllerhas the draw back of limited range RMS output voltage control.

TO CALCULATE THE AVERAGE VALUE (DC VALUE) OF OUTPUTVOLTAGE

$$V_{O(dc)} = \frac{1}{2\pi}\int_{\alpha}^{2\pi} V_m \sin\omega t.d(\omega t)$$

$$V_{O(dc)} = \frac{V_m}{2\pi}\int_{\alpha}^{2\pi} \sin\omega t.d(\omega t)$$

$$V_{O(dc)} = \frac{V_m}{2\pi}\left[-\cos\omega t\Big/_{\alpha}^{2\pi}\right]$$

$$V_{O(dc)} = \frac{V_m}{2\pi}\left[-\cos 2\pi + \cos\alpha\right] \qquad ; \quad \cos 2\pi = 1$$

$$V_{dc} = \frac{V_m}{2\pi}\left[\cos\alpha - 1\right] \quad ; V_m = \sqrt{2}V_S$$

Hence $V_{dc} = \frac{\sqrt{2}V_S}{2\pi}(\cos\alpha - 1)$

When 'α' is varied from 0 to π. V_{dc} varies from 0 to $\frac{-V_m}{\pi}$

DISADVANTAGES OF SINGLE PHASE HALF WAVE AC VOLTAGECONTROLLER.

• The output load voltage has a DC component because the two halves of theoutput voltage waveform are not symmetrical with respect to ‘0’ level. Theinput supply current waveform also has a DC component (average value)which can result in the problem of core saturation of the input supplytransformer.

• The half wave ac voltage controller using a single thyristor and a single diodeprovides control on the thyristor only in one half cycle of the input supply.Hence ac power flow to the load can be controlled only in one half cycle.

• Half wave ac voltage controller gives limited range of RMS output voltagecontrol. Because the RMS value of ac output voltage can be varied from amaximum of 100% of *S V* at a trigger angle α = 0 to a low of 70.7% of *S V* atα =π Radians .

These drawbacks of single phase half wave ac voltage controller can be overcome by using a single phase full wave ac voltage controller.

APPLICATIONS OF RMS VOLTAGE CONTROLLER

• Speed control of induction motor (polyphase ac induction motor).

• Heater control circuits (industrial heating).

• Welding power control.

• Induction heating.

• On load transformer tap changing.

• Lighting control in ac circuits.

• Ac magnet controls.

SINGLE PHASE FULL WAVE AC VOLTAGE CONTROLLER (ACREGULATOR) OR RMS VOLTAGE CONTROLLER WITH RESISTIVELOAD

Single phase full wave ac voltage controller circuit using two SCRs or a singletriac is generally used in most of the ac control applications. The ac power flow to theload can be controlled in both the half cycles by varying the trigger angle 'α ' **.**The RMS value of load voltage can be varied by varying the trigger angle 'α ' **.**The input supply current is alternating in the case of a full wave ac voltage controllerand due to the symmetrical nature of the input supply current waveform there is no dccomponent of input supply current i.e., the average value of the input supply current iszero.

A single phase full wave ac voltage controller with a resistive load is shown inthe figure below. It is possible to control the ac power flow to the load in both the halfcycles by adjusting the trigger angle 'α ' . Hence the full wave ac voltage controller isalso referred to as to a bi-directional controller.

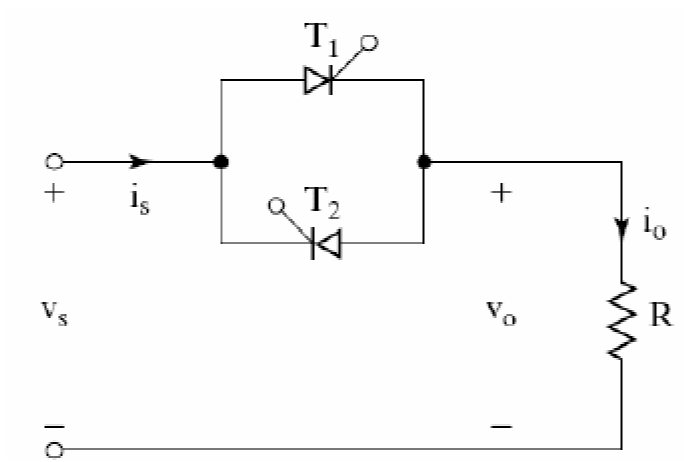


Fig.6: Single phase full wave ac voltage controller (Bi-directional Controller)using SCRs

The thyristor T1 is forward biased during the positive half cycle of the inputsupply voltage. The thyristor T1 is triggered at a delay angle of 'α '(0 ≤α ≤π radians) . Considering the ON thyristor

T1 as an ideal closed switch theinput supply voltage appears across the load resistor RL and the output voltageVO S = v during ωt =α to π radians. The load current flows through the ON thyristorT1 and through the load resistor RL in the downward direction during the conductiontime of T1 from ωt =α to π radians.At ωt =π , when the input voltage falls to zero the thyristor current (which isflowing through the load resistor RL) falls to zero and hence T1 naturally turns off .No current flows in the circuit during ωt =π to (π +α) .The thyristor T2 is forward biased during the negative cycle of input supplyand when thyristor T2 is triggered at a delay angle (π +α) , the output voltage followsthe negative halfcycle of input from ωt = (π +α) to 2π . When T2 is ON, the loadcurrent flows in the reverse direction (upward direction) through T2 duringωt = (π +α) to 2π radians. The time interval (spacing) between the gate triggerpulses of T1 and T2 is kept at π radians or 1800. At ωt = 2π the input supply voltagefalls to zero and hence the load current also falls to zero and thyristor T2 turn offnaturally.

Instead of using two SCR's in parallel, a Triac can be used for full wave acvoltage control.

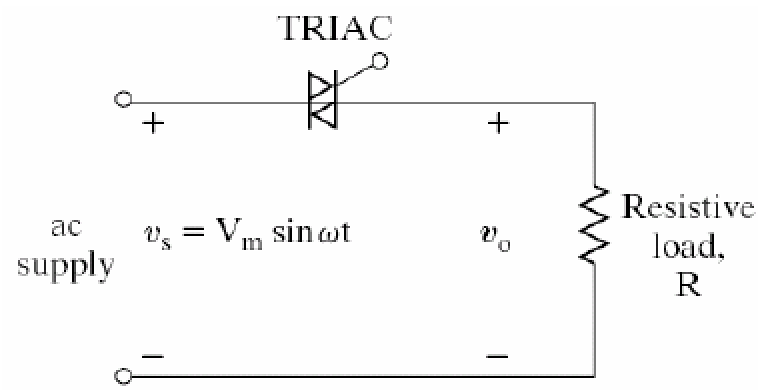


Fig.7: Single phase full wave ac voltage controller (Bi-directional Controller)using TRIAC

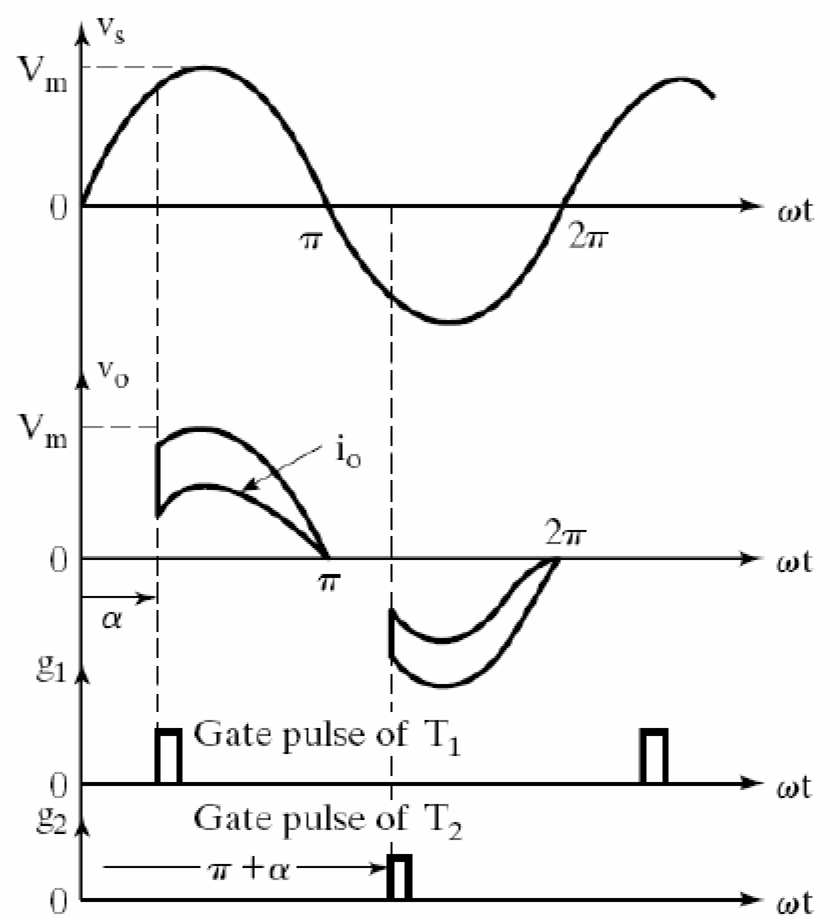


Fig.8 : Waveforms of single phase full wave ac voltage controller

EQUATIONS

Input supply voltage

$$v_S = V_m \sin \omega t = \sqrt{2} V_S \sin \omega t\ ;$$

Output voltage across the load resistor R L **;**

$$v_O = v_L = V_m \sin \omega t\ ;$$

$$\text{for } \omega t = \alpha \text{ to } \pi \text{ and } \omega t = (\pi + \alpha) \text{ to } 2\pi$$

Output load current

$$i_O = \frac{v_O}{R_L} = \frac{V_m \sin \omega t}{R_L} = I_m \sin \omega t\ ;$$

for ωt =α to π and ωt = (π +α) to 2π

TO DERIVE AN EXPRESSION FOR THE RMS VALUE OF OUTPUT(LOAD) VOLTAGE

The RMS value of output voltage (load voltage) can be found using theExpression

$$V_{O(RMS)}^2 = V^2{}_{L(RMS)} = \frac{1}{2\pi}\int_0^{2\pi} v_L{}^2 d\left(\omega t\right);$$

For a full wave ac voltage controller, we can see that the two half cycles ofoutput voltage waveforms are symmetrical and the output pulse time period (or outputpulse repetition time) is π radians. Hence we can also calculate the RMS outputvoltage by using the expression given below.

$$V^2{}_{L(RMS)} = \frac{1}{\pi}\int_0^{\pi} V_m{}^2 \sin^2 \omega t.d\omega t$$

$$V^2{}_{L(RMS)} = \frac{1}{2\pi}\int_0^{2\pi} v_L{}^2 .d\left(\omega t\right);$$

$$v_L = v_O = V_m \sin\omega t \text{ ; For } \omega t = \alpha \text{ to } \pi \text{ and } \omega t = \left(\pi+\alpha\right) \text{ to } 2\pi$$

Hence,

$$V_{L(RMS)}^2 = \frac{1}{2\pi}\left[\int_\alpha^{\pi}\left(V_m \sin\omega t\right)^2 d\left(\omega t\right) + \int_{\pi+\alpha}^{2\pi}\left(V_m \sin\omega t\right)^2 d\left(\omega t\right)\right]$$

$$= \frac{1}{2\pi}\left[V_m{}^2\int_\alpha^{\pi}\sin^2\omega t.d\left(\omega t\right) + V_m{}^2\int_{\pi+\alpha}^{2\pi}\sin^2\omega t.d\left(\omega t\right)\right]$$

$$= \frac{V_m{}^2}{2\pi}\left[\int_\alpha^{\pi}\frac{1-\cos 2\omega t}{2} d\left(\omega t\right) + \int_{\pi+\alpha}^{2\pi}\frac{1-\cos 2\omega t}{2} d\left(\omega t\right)\right]$$

$$=\frac{V_m^{\ 2}}{2\pi\times 2}\left[\int_{\alpha}^{\pi} d\left(\omega t\right)-\int_{\alpha}^{\pi}\cos 2\omega t.d\left(\omega t\right)+\int_{\pi+\alpha}^{2\pi} d\left(\omega t\right)-\int_{\pi+\alpha}^{2\pi}\cos 2\omega t.d\left(\omega t\right)\right]$$

$$=\frac{V_m^{\ 2}}{4\pi}\left[\left(\omega t\right)\Big/_{\alpha}^{\pi}+\left(\omega t\right)\Big/_{\pi+\alpha}^{2\pi}-\left[\frac{\sin 2\omega t}{2}\right]_{\alpha}^{\pi}-\left[\frac{\sin 2\omega t}{2}\right]_{\pi+\alpha}^{2\pi}\right]$$

$$=\frac{V_m^{\ 2}}{4\pi}\left[\left(\pi-\alpha\right)+\left(\pi-\alpha\right)-\frac{1}{2}\left(\sin 2\pi-\sin 2\alpha\right)-\frac{1}{2}\left(\sin 4\pi-\sin 2\left(\pi+\alpha\right)\right)\right]$$

$$=\frac{V_m^{\ 2}}{4\pi}\left[2\left(\pi-\alpha\right)-\frac{1}{2}\left(0-\sin 2\alpha\right)-\frac{1}{2}\left(0-\sin 2\left(\pi+\alpha\right)\right)\right]$$

$$=\frac{V_m^{\ 2}}{4\pi}\left[2\left(\pi-\alpha\right)+\frac{\sin 2\alpha}{2}+\frac{\sin 2\left(\pi+\alpha\right)}{2}\right]$$

$$=\frac{V_m^{\ 2}}{4\pi}\left[2\left(\pi-\alpha\right)+\frac{\sin 2\alpha}{2}+\frac{\sin\left(2\pi+2\alpha\right)}{2}\right]$$

$$=\frac{V_m^{\ 2}}{4\pi}\left[2\left(\pi-\alpha\right)+\frac{\sin 2\alpha}{2}+\frac{1}{2}\left(\sin 2\pi.\cos 2\alpha+\cos 2\pi.\sin 2\alpha\right)\right]$$

$$\sin 2\pi=0 \quad \& \quad \cos 2\pi=1$$

Therefore,

$$V_{L(RMS)}^{2}=\frac{V_m^{\ 2}}{4\pi}\left[2\left(\pi-\alpha\right)+\frac{\sin 2\alpha}{2}+\frac{\sin 2\alpha}{2}\right]$$

$$=\frac{V_m^{\ 2}}{4\pi}\left[2\left(\pi-\alpha\right)+\sin 2\alpha\right]$$

$$V^{2}{}_{L(RMS)}=\frac{V_m^{\ 2}}{4\pi}\left[\left(2\pi-2\alpha\right)+\sin 2\alpha\right]$$

Taking the square root, we get

$$V_{L(RMS)} = \frac{V_m}{2\sqrt{\pi}} \sqrt{\left[(2\pi - 2\alpha) + \sin 2\alpha \right]}$$

$$V_{L(RMS)} = \frac{V_m}{\sqrt{2}\sqrt{2\pi}} \sqrt{\left[(2\pi - 2\alpha) + \sin 2\alpha \right]}$$

$$V_{L(RMS)} = \frac{V_m}{\sqrt{2}} \sqrt{\frac{1}{2\pi} \left[(2\pi - 2\alpha) + \sin 2\alpha \right]}$$

$$V_{L(RMS)} = \frac{V_m}{\sqrt{2}} \sqrt{\frac{1}{2\pi} \left[2\left\{ (\pi - \alpha) + \frac{\sin 2\alpha}{2} \right\} \right]}$$

$$V_{L(RMS)} = \frac{V_m}{\sqrt{2}} \sqrt{\frac{1}{\pi} \left[(\pi - \alpha) + \frac{\sin 2\alpha}{2} \right]}$$

$$V_{L(RMS)} = V_{i(RMS)} \sqrt{\frac{1}{\pi} \left[(\pi - \alpha) + \frac{\sin 2\alpha}{2} \right]}$$

$$V_{L(RMS)} = V_S \sqrt{\frac{1}{\pi} \left[(\pi - \alpha) + \frac{\sin 2\alpha}{2} \right]}$$

Maximum RMS voltage will be applied to the load when $\alpha = 0$, in that case the full sine wave appears across the load. RMS load voltage will be the same as the RMS supply voltage $= \frac{V_m}{\sqrt{2}}$. When α is increased the RMS load voltage decreases.

$$V_{L(RMS)}\Big|_{\alpha=0} = \frac{V_m}{\sqrt{2}} \sqrt{\frac{1}{\pi} \left[(\pi - 0) + \frac{\sin 2 \times 0}{2} \right]}$$

$$V_{L(RMS)}\Big|_{\alpha=0} = \frac{V_m}{\sqrt{2}} \sqrt{\frac{1}{\pi} \left[(\pi) + \frac{0}{2} \right]}$$

$$V_{L(RMS)}\Big|_{\alpha=0} = \frac{V_m}{\sqrt{2}} = V_{i(RMS)} = V_S$$

The output control characteristic for a single phase full wave ac voltagecontroller with resistive load can be obtained by plotting the equation for V $O(RMS)$

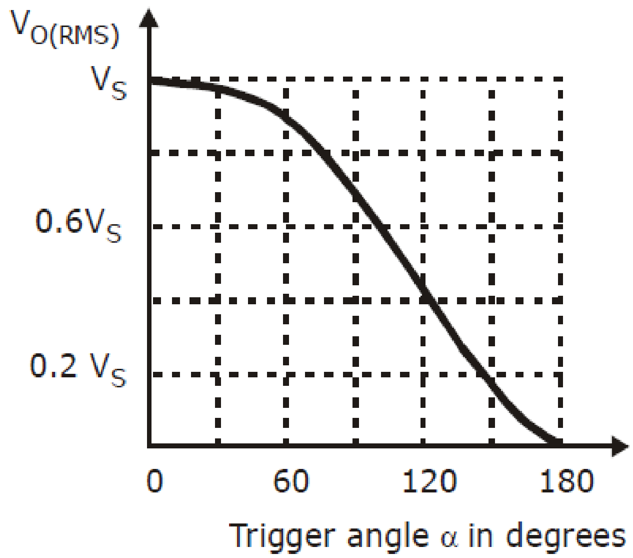
CONTROL CHARACTERISTIC OF SINGLE PHASE FULL-WAVE ACVOLTAGE CONTROLLER WITH RESISTIVE LOAD

The control characteristic is the plot of RMS output voltage V $O(RMS)$ versus thetrigger angle α ; which can be obtained by using the expression for the RMS outputvoltage of a full-wave ac controller with resistive load.

$$V_{O(RMS)} = V_S \sqrt{\frac{1}{\pi}\left[(\pi-\alpha)+\frac{\sin 2\alpha}{2}\right]} \quad ;$$

$$\text{Where} \quad V_S = \frac{V_m}{\sqrt{2}} = \text{RMS value of input supply voltage}$$

Trigger angle α in degrees	**Trigger angle α in radians**	$V_{O(RMS)}$	**%**
0	0	V_S	100% V_S
30^0	$\pi/6$; $(1\pi/6)$	0.985477 V_S	98.54% V_S
60^0	$\pi/3$; $(2\pi/6)$	0.896938 V_S	89.69% V_S
90^0	$\pi/2$; $(3\pi/6)$	0.7071 V_S	70.7% V_S
120^0	$2\pi/3$; $(4\pi/6)$	0.44215 V_S	44.21% V_S
150^0	$5\pi/6$; $(5\pi/6)$	0.1698 V_S	16.98% V_S
180^0	π ; $(6\pi/6)$	0 V_S	0 V_S



We can notice from the figure, that we obtain a much better output controlcharacteristic by using a single phase full wave ac voltage controller. The RMS outputvoltage can be varied from a maximum of 100% *S V* at α = 0 to a minimum of ‘0’ atα =1800 . Thus we get a full range output voltage control by using a single phase fullwave ac voltage controller.

Need For Isolation

In the single phase full wave ac voltage controller circuit using two SCRs orThyristors T1 and T2in parallel, the gating circuits (gate trigger pulse generatingcircuits) of Thyristors T1and T2must be isolated. Figure shows a pulse transformerwith two separate windings to provide isolation between the gating signals of T1 and T2 .

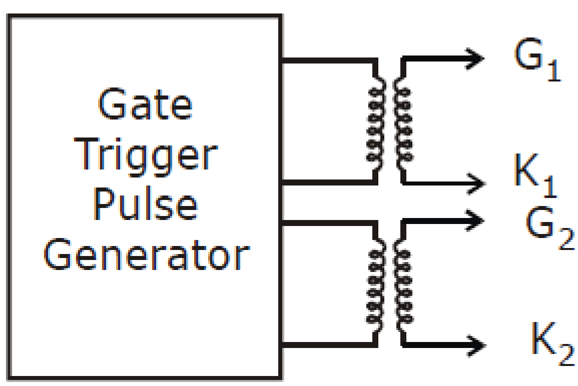
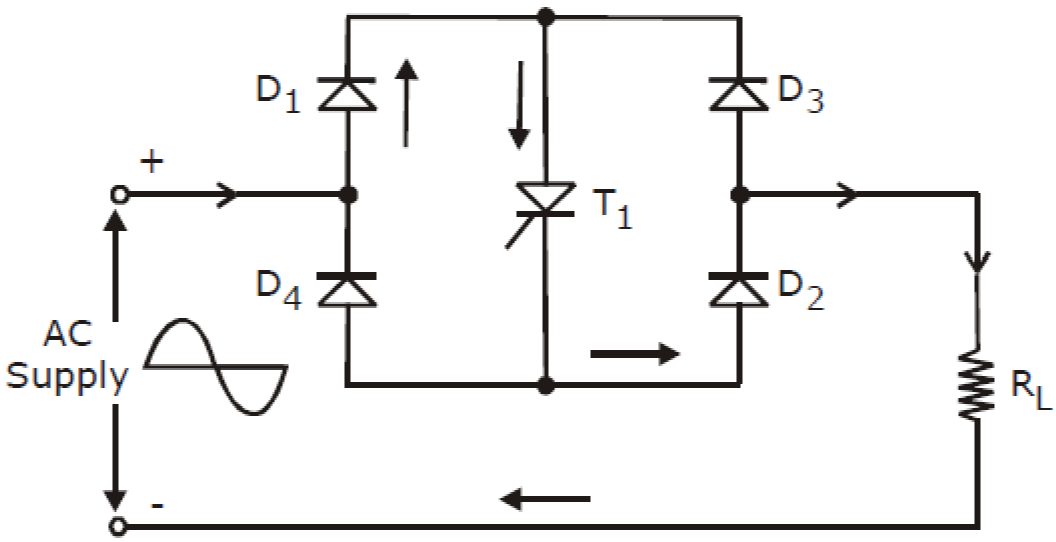


Fig.9 : Pulse Transformer

SINGLE PHASE FULL WAVE AC VOLTAGE CONTROLLER USING ASINGLE THYRISTOR



A single phase full wave ac controller can also be implemented with onethyristor and four diodes connected in a full wave bridge configuration as shown inthe above figure. The four diodes act as a bridge full wave rectifier. The voltageacross the thyristor T1 and current through thyristor T1 are always unidirectional.When $T1$ is triggered at $\omega t = \alpha$, during the positive half cycle ($0 \leq \alpha \leq \pi$) , the loadcurrent flows through D1, T1 , diode D2and through the load. With a resistive load,the thyristor current (flowing through the ON thyristor T1) , the load current falls tozero at $\omega t = \pi$, when the input supply voltage decreases to zero at $\omega t = \pi$, thethyristor naturally turns OFF.In the negative half cycle, diodes 3 4 D &D are forward biased during$\omega t = \pi$ to 2π radians. When T1 is triggered at $\omega t = (\pi + \alpha)$, the load current flows inthe opposite direction (upward direction) through the load, through D3 , T1 and D4 .Thus D3 , D4 and T1 conduct together during the negative half cycle to supply the loadpower. When the input supply voltage becomes zero at $\omega t = 2\pi$, the thyristor current(load current) falls to zero at $\omega t = 2\pi$ and the thyristor 1 T naturally turns OFF. Thewaveforms and the expression for the RMS output voltage are the same as discussedearlier for the single phase full wave ac controller.But however if there is a large inductance in the load circuit, thyristor 1 T maynot be turned OFF at the zero crossing points, in every half cycle of input voltage andthis may result in a loss of output control. This would require detection of the zerocrossing of the load current waveform in order to ensure guaranteed turn off of theconducting thyristor before triggering the thyristor in the next half cycle, so that wegain control on the output voltage.

In this full wave ac controller circuit using a single thyristor, as there are threepower devices conducting together at the same time there is more conduction voltagedrop and an increase in the ON state conduction losses and hence efficiency is alsoreduced.The diode bridge rectifier and thyristor (or a power transistor) act together as abidirectional switch which is commercially available as a single device module and it has relatively low ON state conduction loss. It can be used for bidirectional loadcurrent control and for controlling the RMS output voltage.

SINGLE PHASE FULL WAVE AC VOLTAGE CONTROLLER(BIDIRECTIONAL CONTROLLER) WITH RL LOAD

In this section we will discuss the operation and performance of a single phasefull wave ac voltage controller with RL load. In practice most of the loads are of RLtype. For example if we consider a single phase full wave ac voltage controllercontrolling the speed of a single phase ac induction motor, the load which is theinduction motor winding is an RL type of load, where R represents the motor windingresistance and L represents the motor winding inductance.A single phase full wave ac voltage controller circuit (bidirectional controller)with an RL load using two thyristors T1 and T2 (T1 and T2 are two SCRs) connected inparallel is shown in the figure below. In place of two thyristors a single Triac can beused to implement a full wave ac controller, if a suitable Traic is available for thedesired RMS load current and the RMS output voltage ratings.

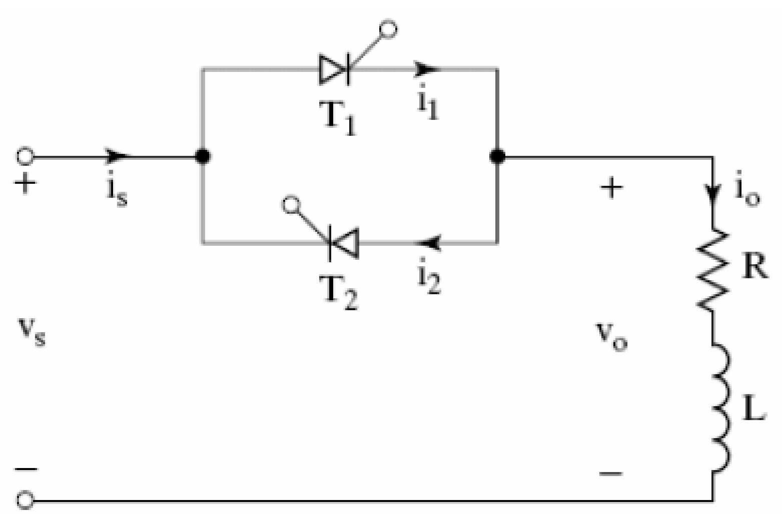


Fig.10 : Single phase full wave ac voltage controller with RL load

The thyristor T1 is forward biased during the positive half cycle of inputsupply. Let us assume that *T*1 is triggered at $\omega t = \alpha$, by applying a suitable gatetrigger pulse to T1 during the positive half cycle of input supply. The output voltageacross the load follows the input supply voltage when T1 is ON. The load current I*O* flows through the thyristor T1 and through the load in the downward direction. Thisload current pulse flowing through T1 can be considered as the positive current pulse.Due to the inductance in the load, the load current I*O* flowing through T1 would notfall to zero at $\omega t = \pi$, when the input supply voltage starts to become negative.The thyristor T1 will continue to conduct the load current until all the inductiveenergy stored in the load inductor L is completely utilized and the load current through T1 falls to zero at $\omega t = \beta$, where β is referred to as the Extinction angle,(the value of ωt) at which the load current falls to zero. The extinction angle β ismeasured from the point of the beginning of the positive half cycle of input supply tothe point where the load current falls to zero.The thyristor T1 conducts from $\omega t = \alpha$ to β . The conduction angle of T1 is$\delta = (\beta - \alpha)$, which depends on the delay angle α and the load impedance angle φ .The waveforms of the input supply voltage, the gate trigger pulses of T1 and T2 , thethyristor current, the load current and the load voltage waveforms appear as shown inthe figure below.

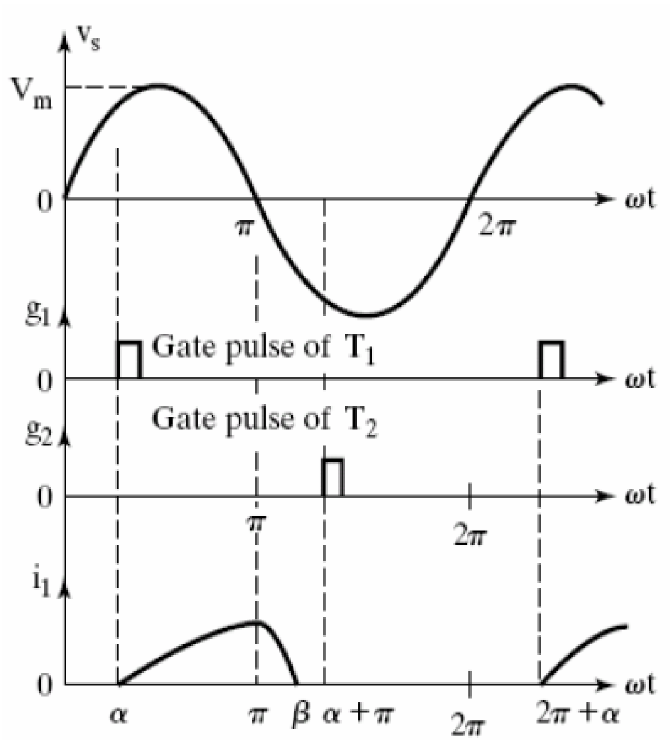


Fig. 11 : Input supply voltage & Thyristor current waveforms

β is the extinction angle which depends upon the load inductance value.

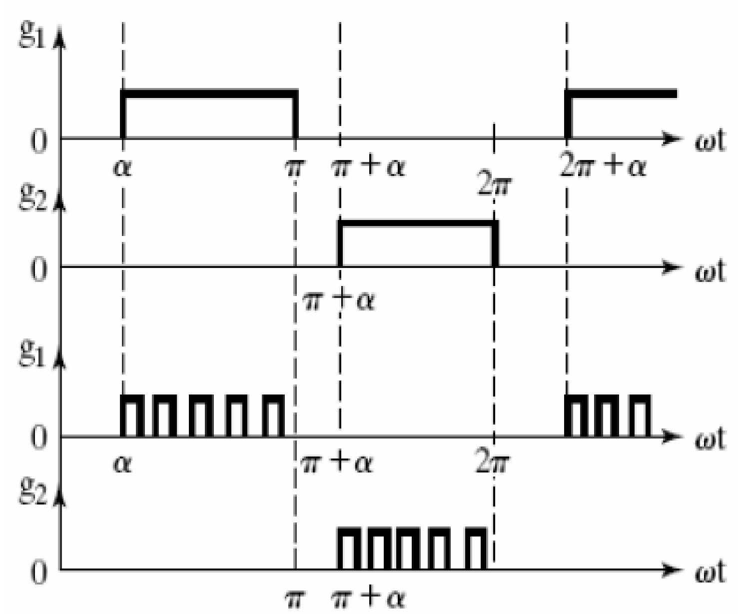
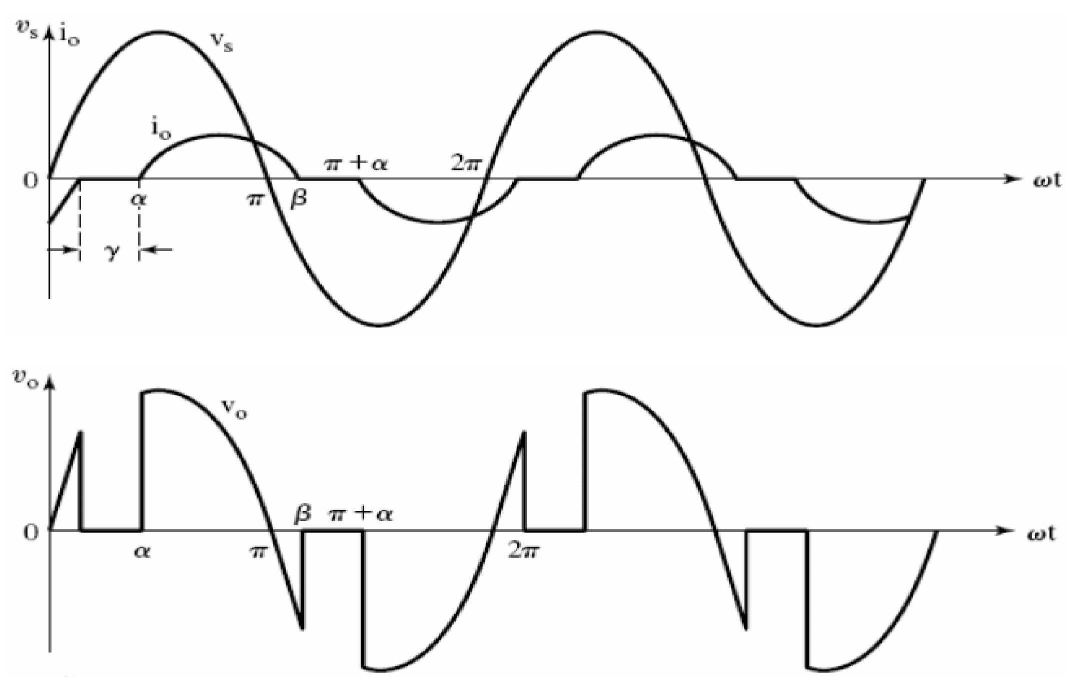


Fig. 12 : Gating Signals

Waveforms of single phase full wave ac voltage controller with RL load for $\alpha > \varphi$ **. Discontinuous load current operation occurs for** $\alpha > \varphi$ **and** $\beta < (\pi + \alpha)$ **;i.e.,** $(\beta - \alpha) < \pi$ **, conduction angle** $< \pi$ **.**



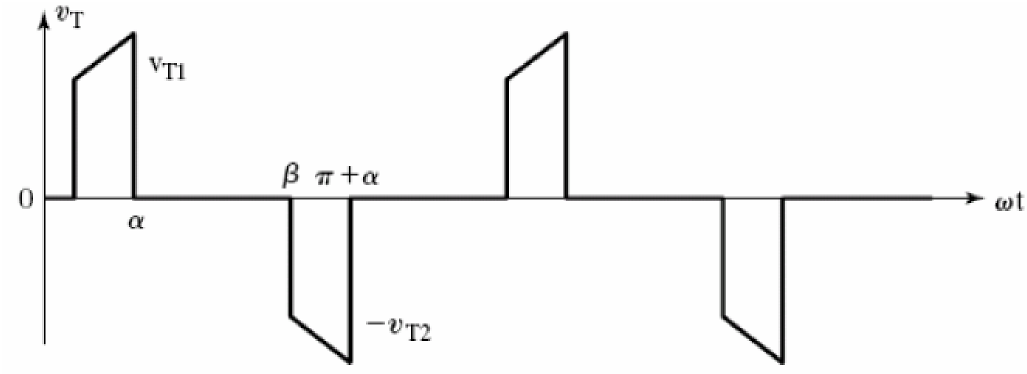
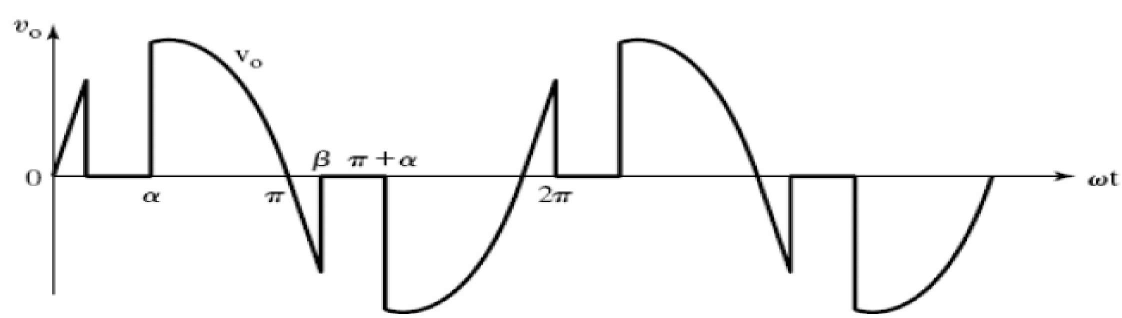


Fig. 13 : Waveforms of Input supply voltage, Load Current, Load Voltage andThyristor Voltage across T1

Note

- The RMS value of the output voltage and the load current may be varied byvarying the trigger angle α .
- This circuit, AC RMS voltage controller can be used to regulate the RMSvoltage across the terminals of an ac motor (induction motor). It can be used tocontrol the temperature of a furnace by varying the RMS output voltage.
- For very large load inductance 'L' the SCR may fail to commutate, after it istriggered and the load voltage will be a full sine wave (similar to the appliedinput supply voltage and the output control will be lost) as long as the gatingsignals are applied to the thyristors T1 and T2 . The load current waveform willappear as a full continuous sine wave and the load current waveform lagsbehind the output sine wave by the load power factor angle φ.

TO DERIVE AN EXPRESSION FOR RMS OUTPUT VOLTAGE V$_{O(RMS)}$ **OF ASINGLE PHASE FULL-WAVE AC VOLTAGE CONTROLLER WITH RL LOAD.**



When $\alpha > \phi$, the load current and load voltage waveforms become discontinuous as shown in the figure above.

$$V_{O(RMS)} = \left[\frac{1}{\pi} \int_{\alpha}^{\beta} V_m^{\ 2} \sin^2 \omega t . d(\omega t) \right]^{\frac{1}{2}}$$

Output $v_o = V_m \sin \omega t$, for $\omega t = \alpha$ to β, when T_1 is ON.

$$V_{O(RMS)} = \left[\frac{V_m^{\ 2}}{\pi} \int_{\alpha}^{\beta} \frac{(1-\cos 2\omega t)}{2} d(\omega t) \right]^{1/2}$$

$$V_{O(RMS)} = \left[\frac{V_m^{\ 2}}{2\pi} \left\{ \int_{\alpha}^{\beta} d(\omega t) - \int_{\alpha}^{\beta} \cos 2\omega t . d(\omega t) \right\} \right]^{1/2}$$

$$V_{O(RMS)} = \left[\frac{V_m^{\ 2}}{2\pi} \left\{ (\omega t) \Big/_{\alpha}^{\beta} - \left(\frac{\sin 2\omega t}{2} \right) \Big/_{\alpha}^{\beta} \right\} \right]^{1/2}$$

$$V_{O(RMS)} = \left[\frac{V_m^{\ 2}}{2\pi} \left\{ (\beta - \alpha) - \frac{\sin 2\beta}{2} + \frac{\sin 2\alpha}{2} \right\} \right]^{1/2}$$

$$V_{O(RMS)} = V_m \left[\frac{1}{2\pi} \left\{ (\beta - \alpha) + \frac{\sin 2\alpha}{2} - \frac{\sin 2\beta}{2} \right\} \right]^{1/2}$$

$$V_{O(RMS)} = \frac{V_m}{\sqrt{2}} \left[\frac{1}{\pi} \left\{ (\beta - \alpha) + \frac{\sin 2\alpha}{2} - \frac{\sin 2\beta}{2} \right\} \right]^{1/2}$$

The RMS output voltage across the load can be varied by changing the trigger angle α .

For a purely resistive load $L = 0$, therefore load power factor angle $\phi = 0$.

$$\phi = \tan^{-1}\left(\frac{\omega L}{R} \right) = 0 \quad ;$$

Extinction angle $\beta = \pi$ radians $= 180^0$

PERFORMANCE PARAMETERS OF A SINGLE PHASE FULL WAVE ACVOLTAGE CONTROLLER WITH RESISTIVE LOAD

- **RMS Output Voltage** $V_{O(RMS)} = \frac{V_m}{\sqrt{2}}\sqrt{\frac{1}{\pi}\left[(\pi-\alpha)+\frac{\sin 2\alpha}{2}\right]}$; $\frac{V_m}{\sqrt{2}} = V_S$ = RMS input supply voltage.

- $I_{O(RMS)} = \frac{V_{O(RMS)}}{R_L}$ = RMS value of load current.

- $I_S = I_{O(RMS)}$ = RMS value of input supply current.

• **Output load power**

$$P_O = I_{O(RMS)}^2 \times R_L$$

• **Input Power Factor**

$$PF = \frac{P_O}{V_S \times I_S} = \frac{I_{O(RMS)}^2 \times R_L}{V_S \times I_{O(RMS)}} = \frac{I_{O(RMS)} \times R_L}{V_S}$$

$$PF = \frac{V_{O(RMS)}}{V_S} = \sqrt{\frac{1}{\pi}\left[(\pi-\alpha)+\frac{\sin 2\alpha}{2}\right]}$$

• **Average Thyristor Current**

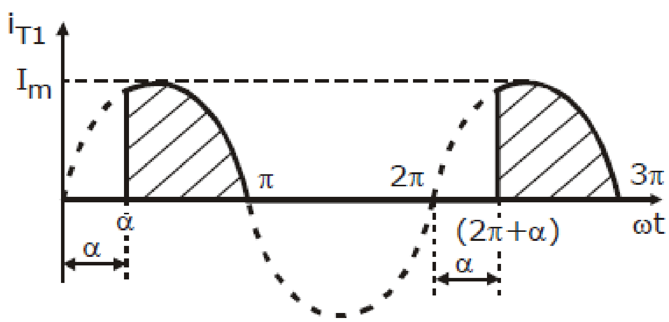


Fig.14 : Thyristor Current Waveform

$$I_{T(Avg)} = \frac{1}{2\pi}\int_{\alpha}^{\pi} i_T d(\omega t) = \frac{1}{2\pi}\int_{\alpha}^{\pi} I_m \sin\omega t.d(\omega t)$$

$$I_{T(Avg)} = \frac{I_m}{2\pi}\int_{\alpha}^{\pi} \sin\omega t.d(\omega t) = \frac{I_m}{2\pi}\left[-\cos\omega t \Big/_{\alpha}^{\pi}\right]$$

$$I_{T(Avg)} = \frac{I_m}{2\pi}\left[-\cos\pi + \cos\alpha\right] = \frac{I_m}{2\pi}\left[1 + \cos\alpha\right]$$

• **Maximum Average Thyristor Current, for** $\alpha = 0$,

$$I_{T(Avg)} = \frac{I_m}{\pi}$$

RMS Thyristor Current

$$I_{T(RMS)} = \sqrt{\frac{1}{2\pi}\left[\int_{\alpha}^{\pi} I_m^2 \sin^2\omega t.d(\omega t)\right]}$$

$$I_{T(RMS)} = \frac{I_m}{\sqrt{2}}\sqrt{\frac{1}{2\pi}\left[(\pi-\alpha) + \frac{\sin 2\alpha}{2}\right]}$$

• **Maximum RMS Thyristor Current, for** $\alpha = 0$,

$$I_{T(RMS)} = \frac{I_m}{2}$$

In the case of a single phase full wave ac voltage controller circuit using aTriac with resistive load, the average thyristor current I*T Avg* = . Because the Triacconducts in both the half cycles and the thyristor current is alternating and we obtain asymmetrical thyristor current waveform which gives an average value of zero onintegration.

PERFORMANCE PARAMETERS OF A SINGLE PHASE FULL WAVE ACVOLTAGE CONTROLLER WITH R-L LOAD

The Expression for the Output (Load) Current

The expression for the output (load) current which flows through the thyristor, during $\omega t = \alpha$ to β is given by

$$i_O = i_{T_1} = \frac{V_m}{Z}\left[\sin(\omega t - \phi) - \sin(\alpha - \phi)e^{\frac{-R}{\omega L}(\omega t - \alpha)}\right] \quad ; \quad \text{for } \alpha \le \omega t \le \beta$$

Where,

$V_m = \sqrt{2}V_S$ = Maximum or peak value of input ac supply voltage.

$Z = \sqrt{R^2 + (\omega L)^2}$ = Load impedance.

$\phi = \tan^{-1}\left(\frac{\omega L}{R}\right)$ = Load impedance angle (load power factor angle).

α = Thyristor trigger angle = Delay angle.

β = Extinction angle of thyristor, (value of ωt) at which the thyristor (load) current falls to zero.

β is calculated by solving the equation

$$\sin(\beta - \phi) = \sin(\alpha - \phi)e^{\frac{-R}{\omega L}(\beta - \alpha)}$$

Thyristor Conduction Angle $\delta = (\beta - \alpha)$

Maximum thyristor conduction angle $\delta = (\beta - \alpha) = \pi$ radians = 180^0 for $\alpha \le \phi$.

RMS Output Voltage

$$V_{O(RMS)} = \frac{V_m}{\sqrt{2}}\sqrt{\frac{1}{\pi}\left[(\beta - \alpha) + \frac{\sin 2\alpha}{2} - \frac{\sin 2\beta}{2}\right]}$$

The Average Thyristor Current

$$I_{T(Avg)} = \frac{1}{2\pi}\left[\int_{\alpha}^{\beta} i_{T_1} d(\omega t)\right]$$

$$I_{T(Avg)} = \frac{1}{2\pi}\left[\int_{\alpha}^{\beta} \frac{V_m}{Z}\left[\sin(\omega t-\phi)-\sin(\alpha-\phi)e^{\frac{-R}{\omega L}(\omega t-\alpha)}\right]d(\omega t)\right]$$

$$I_{T(Avg)} = \frac{V_m}{2\pi Z}\left[\int_{\alpha}^{\beta} \sin(\omega t-\phi).d(\omega t)-\int_{\alpha}^{\beta}\sin(\alpha-\phi)e^{\frac{-R}{\omega L}(\omega t-\alpha)}d(\omega t)\right]$$

Maximum value of $I_{T(Avg)}$ occur at $\alpha = 0$. The thyristors should be rated for maximum $I_{T(Avg)} = \left(\frac{I_m}{\pi}\right)$, where $I_m = \frac{V_m}{Z}$.

RMS Thyristor Current I $T\,(RMS\,)$

$$I_{T(RMS)} = \sqrt{\left[\frac{1}{2\pi}\int_{\alpha}^{\beta} i_{T_1}^2 d(\omega t)\right]}$$

Maximum value of $I_{T(RMS)}$ occurs at $\alpha = 0$. Thyristors should be rated for maximum $I_{T(RMS)} = \left(\frac{I_m}{2}\right)$

When a Triac is used in a single phase full wave ac voltage controller with RL type of load, then $I_{T(Avg)} = 0$ and maximum $I_{T(RMS)} = \frac{I_m}{\sqrt{2}}$

PROBLEMS

1. *A single phase full wave ac voltage controller supplies an RL load. The input supply voltage is 230V, RMS at 50Hz. The load has L = 10mH, R = 10Ω, the delay angle of thyristors T_1 and T_2 are equal, where $\alpha_1 = \alpha_2 = \frac{\pi}{3}$. Determine*
    a. *Conduction angle of the thyristor T_1.*
    b. *RMS output voltage.*
    c. *The input power factor.*
    Comment on the type of operation.

    Given

    $V_s = 230V$, $f = 50Hz$, $L = 10mH$, $R = 10\Omega$, $\alpha = 60^0$,

    $\alpha = \alpha_1 = \alpha_2 = \frac{\pi}{3}$ radians, .

$$V_m = \sqrt{2}V_S = \sqrt{2} \times 230 = 325.2691193 \ V$$

$$Z = \text{ Load Impedance} = \sqrt{R^2 + (\omega L)^2} = \sqrt{(10)^2 + (\omega L)^2}$$

$$\omega L = (2\pi fL) = \left(2\pi \times 50 \times 10 \times 10^{-3}\right) = \pi = 3.14159\Omega$$

$$Z = \sqrt{(10)^2 + (3.14159)^2} = \sqrt{109.8696} = 10.4818\Omega$$

$$I_m = \frac{V_m}{Z} = \frac{\sqrt{2} \times 230}{10.4818} = 31.03179 \ A$$

Load Impedance Angle $\phi = \tan^{-1}\left(\frac{\omega L}{R}\right)$

$$\phi = \tan^{-1}\left(\frac{\pi}{10}\right) = \tan^{-1}(0.314159) = 17.44059^0$$

Trigger Angle $\alpha > \phi$. Hence the type of operation will be discontinuous load current operation, we get

$$\beta < (\pi + \alpha)$$

$$\beta < (180 + 60)\ ;\ \beta < 240^0$$

Therefore the range of β is from 180 degrees to 240 degrees. $(180^0 < \beta < 240^0)$

Extinction Angle β is calculated by using the equation

$$\sin(\beta - \phi) = \sin(\alpha - \phi)e^{\frac{-R}{\omega L}(\beta - \alpha)}$$

In the exponential term the value of α and β should be substituted inradians. Hence

$$\sin(\beta-\phi)=\sin(\alpha-\phi)e^{\frac{-R}{\omega L}(\beta_{Rad}-\alpha_{Rad})} \quad ; \quad \alpha_{Rad}=\left(\frac{\pi}{3}\right)$$

$$(\alpha-\phi)=(60-17.44059)=42.5594^0$$

$$\sin(\beta-17.44)^0=\sin\left(42.5594^0\right)e^{\frac{-10}{\pi}(\beta-\alpha)}$$

$$\sin(\beta-17.44)^0=0.676354e^{-3.183(\beta-\alpha)}$$

$$180^0 \rightarrow \pi \text{ radians}, \quad \beta_{Rad}=\frac{\beta^0\times\pi}{180^0}$$

Assuming $\beta=190^0$;

$$\beta_{Rad}=\frac{\beta^0\times\pi}{180^0}=\frac{190^0\times\pi}{180}=3.3161$$

$$\text{L.H.S: } \sin(190-17.44)^0=\sin(172.56)=0.129487$$

$$\text{R.H.S: } 0.676354\times e^{-3.183\left(3.3161-\frac{\pi}{3}\right)}=4.94\times10^{-4}$$

Assuming $\beta=183^0$;

$$\beta_{Rad}=\frac{\beta^0\times\pi}{180^0}=\frac{183^0\times\pi}{180}=3.19395$$

$$(\beta-\alpha)=\left(3.19395-\frac{\pi}{3}\right)=2.14675$$

$$\text{L.H.S: } \sin(\beta-\phi)=\sin(183-17.44)=\sin 165.56^0=0.24936$$

$$\text{R.H.S: } 0.676354e^{-3.183(2.14675)} \quad =7.2876\times10^{-4}$$

Assuming $\beta \approx 180^0$

$$\beta_{Rad}=\frac{\beta^0\times\pi}{180^0}=\frac{180^0\times\pi}{180}=\pi$$

$$(\beta-\alpha)=\left(\pi-\frac{\pi}{3}\right)=\left(\frac{2\pi}{3}\right)$$

$$\text{L.H.S: } \sin(\beta-\phi)=\sin(180-17.44)=0.2997$$

$$\text{R.H.S: } 0.676354e^{-3.183\left(\pi-\frac{\pi}{3}\right)}=8.6092\times10^{-4}$$

Assuming $\beta=196^0$

$$\beta_{Rad}=\frac{\beta^0\times\pi}{180^0}=\frac{196^0\times\pi}{180}=3.420845$$

$$\text{L.H.S: } \sin(\beta-\phi)=\sin(196-17.44)=0.02513$$

$$\text{R.H.S: } 0.676354e^{-3.183\left(3.420845-\frac{\pi}{3}\right)}=3.5394\times10^{-4}$$

Assuming $\beta=197^0$

$$\beta_{Rad}=\frac{\beta^0\times\pi}{180^0}=\frac{197^0\times\pi}{180}=3.43829$$

$$\text{L.H.S: } \sin(\beta-\phi)=\sin(197-17.44)=7.69=7.67937\times10^{-3}$$

$$\text{R.H.S: } 0.676354e^{-3.183\left(3.43829-\frac{\pi}{3}\right)}=4.950386476\times10^{-4}$$

$Assuming\ \ \beta = 197.42^0$

$$\beta_{Rad} = \frac{\beta^0 \times \pi}{180^0} = \frac{197.42 \times \pi}{180} = 3.4456$$

L.H.S: $\sin(\beta - \phi) = \sin(197.42 - 17.44) = 3.4906 \times 10^{-4}$

R.H.S: $0.676354 e^{-3.183\left(3.4456 - \frac{\pi}{3}\right)} = 3.2709 \times 10^{-4}$

Conduction Angle $\delta = (\beta - \alpha) = \left(197.42^0 - 60^0\right) = 137.42^0$

RMS Output Voltage

$$V_{O(RMS)} = V_S \sqrt{\frac{1}{\pi}\left[(\beta - \alpha) + \frac{\sin 2\alpha}{2} - \frac{\sin 2\beta}{2}\right]}$$

$$V_{O(RMS)} = 230 \sqrt{\frac{1}{\pi}\left[\left(3.4456 - \frac{\pi}{3}\right) + \frac{\sin 2\left(60^0\right)}{2} - \frac{\sin 2\left(197.42^0\right)}{2}\right]}$$

$$V_{O(RMS)} = 230 \sqrt{\frac{1}{\pi}\left[(2.39843) + 0.4330 - 0.285640\right]}$$

$$V_{O(RMS)} = 230 \times 0.9 = 207.0445 \text{ V}$$

Input Power Factor

$$PF = \frac{P_O}{V_S \times I_S}$$

$$I_{O(RMS)} = \frac{V_{O(RMS)}}{Z} = \frac{207.0445}{10.4818} = 19.7527 \text{ A}$$

$$P_O = I_{O(RMS)}^2 \times R_L = (19.7527)^2 \times 10 = 3901.716 \text{ W}$$

$$V_S = 230V, \qquad I_S = I_{O(RMS)} = 19.7527$$

$$PF = \frac{P_O}{V_S \times I_S} = \frac{3901.716}{230 \times 19.7527} = 0.8588$$

2. A single phase half wave ac regulator using one SCR in anti-parallel with adiode feeds 1 kW, 230 V heater. Find load power for a firing angle of 45^0.

Solution

$$\alpha = 45^0 = \frac{\pi}{4}, \quad V_S = 230 \text{ V} \; ; \; P_O = 1KW = 1000W$$

At standard rms supply voltage of 230V, the heater dissipates 1KW of outputpower

Therefore

$$P_O = V_O \times I_O = \frac{V_O \times V_O}{R} = \frac{V_O^2}{R}$$

Resistance of heater

$$R = \frac{V_O^2}{P_O} = \frac{(230)^2}{1000} = 52.9\Omega$$

RMS value of output voltage

$$V_O = V_S\left[\frac{1}{2\pi}\left(2\pi - \alpha + \frac{\sin 2\alpha}{2}\right)\right]^{\frac{1}{2}} \qquad ; \text{ for firing angle } \alpha = 45^0$$

$$V_O = 230\left[\frac{1}{2\pi}\left(2\pi - \frac{\pi}{4} + \frac{\sin 90}{2}\right)\right]^{\frac{1}{2}} = 224.7157 \text{ Volts}$$

RMS value of output current

$$I_O = \frac{V_O}{R} = \frac{224.9}{52.9} = 4.2479 \text{ Amps}$$

Load Power

$$P_O = I_O^2 \times R = (4.25)^2 \times 52.9 = 954.56 \text{ Watts}$$

UNIT – III

Single Phase Bridge Converter And Harmonic Analysis Fully Controlled Converters

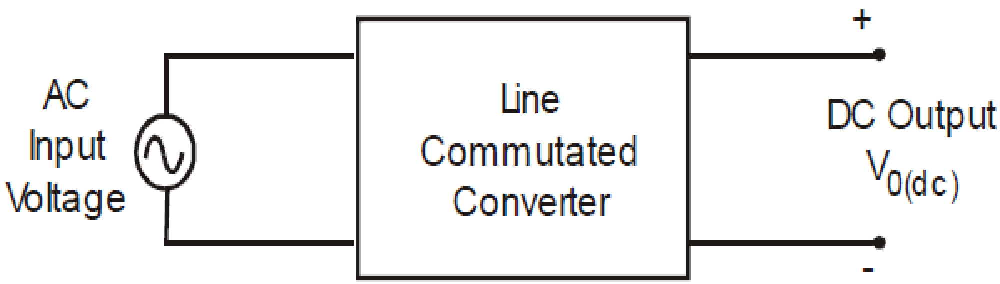
UNIT - III

Single Phase Bridge Converter andHarmonic Analysis Fully ControlledConverters

(Line Commutated AC to DC converters)

INTRODUCTION TO CONTROLLED RECTIFIERS

Controlled rectifiers are line commutated ac to dc power converters which are used to convert a fixed voltage, fixed frequency ac power supply into variable dc output voltage.



Type of input: Fixed voltage, fixed frequency ac power supply.
Type of output: Variable dc output voltage

The input supply fed to a controlled rectifier is ac supply at a fixed rms voltage and at a fixed frequency. We can obtain variable dc output voltage by using controlled rectifiers. By employing phase controlled thyristors in the controlled rectifier circuits we can obtain variable dc output voltage and variable dc (average) output current by varying the trigger angle (phase angle) at which the thyristors are triggered. We obtain a uni-directional and pulsating load current waveform, which has a specific average value.

The thyristors are forward biased during the positive half cycle of input supply and can be turned ON by applying suitable gate trigger pulses at the thyristor gate leads. The thyristor current and the load current begin to flow once the thyristors are triggered (turned ON) say at $\omega t = \alpha$. The load current flows when the thyristors conduct from $\omega t = \alpha$ to β. The output voltage across the load follows the input supply voltage through the conducting thyristor. At $\omega t = \beta$, when the load current falls to zero, the thyristors turn off due to AC line (natural) commutation.

In some bridge controlled rectifier circuits the conducting thyristor turns off, when the other thyristor is (other group of thyristors are) turned ON. The thyristor remains reverse biased during the negative half cycle of input supply. The type of commutation used in controlled rectifier circuits is referred to AC line commutation or Natural commutation or AC phase commutation.

When the input ac supply voltage reverses and becomes negative during the negative half cycle, the thyristor becomes reverse biased and hence turns off. There are several types of power converters which use ac line commutation. These are referred to as line commutated converters. Different types of line commutated converters are

- Phase controlled rectifiers which are AC to DC converters.
- AC to AC converters
- AC voltage controllers, which convert input ac voltage intovariable ac output voltage at the same frequency.
- Cyclo converters, which give low output frequencies.

All these power converters operate from ac power supply at a fixed rms input supply voltage and at a fixed input supply frequency. Hence they use ac line commutation for turning off the thyristors after they have been triggered ON by the gating signals.

DIFFERENCES BETWEEN DIODE RECTIFIERS AND PHASE CONTROLLED RECTIFIERS

The diode rectifiers are referred to as uncontrolled rectifiers which make use of power semiconductor diodes to carry the load current. The diode rectifiers give a fixed dc output voltage (fixed average output voltage) and each diode rectifying element conducts for one half cycle duration (T/2 seconds), that is the diode conduction angle = 180^0 or π radians.

A single phase half wave diode rectifier gives (under ideal conditions) an average dc output voltage $V_{O(dc)} = \frac{V_m}{\pi}$ and single phase full wave diode rectifier gives (under ideal conditions) an average dc output voltage $V_{O(dc)} = \frac{2V_m}{\pi}$ where V_m is maximum value of the available ac supply voltage.

Thus we note that we can not control (we can not vary) the dc output voltage or the average dc load current in a diode rectifier circuit.

In a phase controlled rectifier circuit we use a high current and a high power thyristor device (silicon controlled rectifier; SCR) for conversion of ac input power into dc output power.

Phase controlled rectifier circuits are used to provide a variable voltage output dc and a variable dc (average) load current.

We can control (we can vary) the average value (dc value) of the output load voltage (and hence the average dc load current) by varying the thyristor trigger angle.

We can control the thyristor conduction angle δ from $180^0$to $0^0$by varying the trigger angle α from $0^0$to 180^0, where thyristor conduction angle $\delta = (\pi - \alpha)$

APPLICATIONS OF PHASE CONTROLLED RECTIFIERS

- DC motor control in steel mills, paper and textile mills employing dc motordrives.
- AC fed traction system using dc traction motor.
- Electro-chemical and electro-metallurgical processes.
- Magnet power supplies.
- Reactor controls.
- Portable hand tool drives.
- Variable speed industrial drives.
- Battery charges.
- High voltage DC transmission.
- Uninterruptible power supply systems (UPS).

Some years back ac to dc power conversion was achieved using motor generator sets, mercury arc rectifiers, and thyratorn tubes. The modern ac to dc power converters are designed using high power, high current thyristors and presently most of the ac-dc power converters are thyristorised power converters. The thyristor devices are phase controlled to obtain a variable dc output voltage across the output load terminals. The phase controlled thyristor converter uses ac line commutation (natural commutation) for commutating (turning off) the thyristors that have been turned ON.

The phase controlled converters are simple and less expensive and are widely used in industrial applications for industrial dc drives. These converters are classified as two quadrant converters if the output voltage can be made either positive or negative for a given polarity of output load current. There are also single quadrant acdc converters where the output voltage is only positive and cannot be made negative for a given polarity of output current. Of course single quadrant converters can also be designed to provide only negative dc output voltage.

The two quadrant converter operation can be achieved by using fully controlled bridge converter circuit and for single quadrant operation we use a half controlled bridge converter.

CLASSIFICATION OF PHASE CONTROLLED RECTIFIERS

The phase controlled rectifiers can be classified based on the type of input power supply as

- Single Phase Controlled Rectifiers which operate from single phase ac input power supply.
- Three Phase Controlled Rectifiers which operate from three phase ac input power supply.

DIFFERENT TYPES OF SINGLE PHASE CONTROLLED RECTIFIERS

*Single Phase Controlled Rectifier*s are further subdivided into different types

- *Half wave controlled rectifier* which uses a single thyristor device (which provides output control only in one half cycle of input ac supply, and it provides low dc output).
- *Full wave controlled rectifiers* (which provide higher dc output)

I. Full wave controlled rectifier using a center tapped transformer (which requires two thyristors).
II. Full wave bridge controlled rectifiers (which do not require a center tapped transformer)

- *Single phase semi-converter* (half controlled bridge converter, using two SCR's and two diodes, to provide single quadrant operation).
- *Single phase full converter* (fully controlled bridge converter which requires four SCR's, to provide two quadrant operation).

*Three Phase Controlled Rectifier*s are of different types

- Three phase half wave controlled rectifiers.
- Three phase full wave controlled rectiriers.

I. Semi converter (half controlled bridge converter).

II. Full converter (fully controlled bridge converter).

PRINCIPLE OF PHASE CONTROLLED RECTIFIER OPERATION

The basic principle of operation of a phase controlled rectifier circuit is explained with reference to a single phase half wave phase controlled rectifier circuit with a resistive load shown in the figure.

A single phase half wave thyristor converter which is used for ac-dc power conversion is shown in the above figure. The input ac supply is obtained from a main supply transformer to provide the desired ac supply voltage to the thyristor converter depending on the output dc voltage required. $_P v$ represents the primary input ac supply voltage. $_S v$ represents the secondary ac supply voltage which is the output of the transformer secondary.

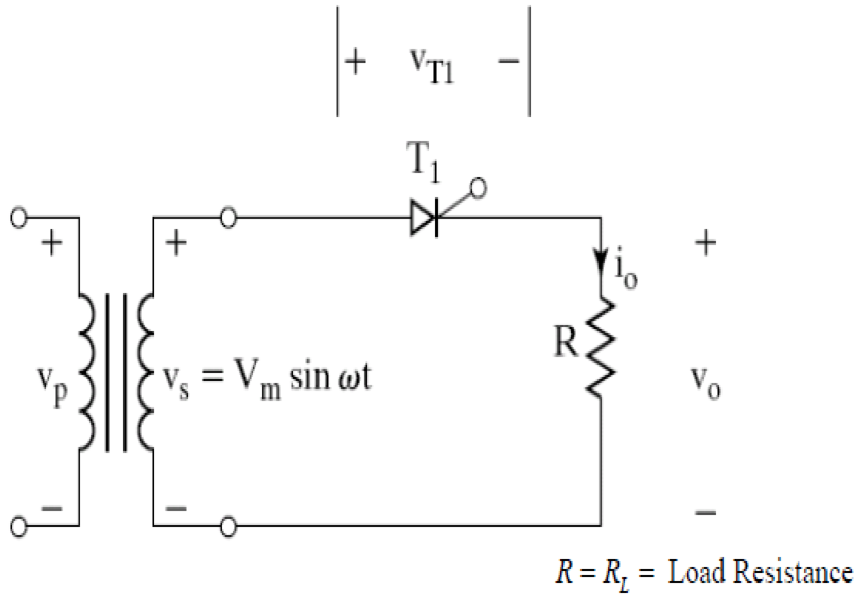


Fig.1 Single Phase Half-Wave Thyristor Converter with a Resistive Load

During the positive half cycle of input supply when the upper end of the transformer secondary is at a positive potential with respect to the lower end, the thyristor anode is positive with respect to its cathode and the thyristor is in a forward biased state. The thyristor is triggered at a delay angle of $\omega t = \alpha$, by applying a suitable gate trigger pulse to the gate lead of thyristor. When the thyristor is triggered at a delay angle of $\omega t = \alpha$, the thyristor conducts and assuming an ideal thyristor, the thyristor behaves as a closed switch and the input supply voltage appears across the load when the thyristor conducts from $\omega t = \alpha$ to π radians. Output voltage $V_O = V_S$ when the thyristor conducts from $\omega t = \alpha$ to π.

For a purely resistive load, the load current i_o (output current) that flows when the thyristor T_1 is on, is given by the expression

$$i_o = \frac{V_O}{R_L}, for\ \alpha \le \omega t \le \pi$$

The output load current waveform is similar to the output load voltage waveform during the thyristor conduction time from α to π. The output current and the output voltage waveform are in phase for a resistive load. The load current increases as the input supply voltage increases and the maximum load current flows at $= \frac{\pi}{2}$, when the input supply voltage is at its maximum value.

The maximum value (peak value) of the load current is calculated as

$$i_{o(max)} = I_m = \frac{V_m}{R_L}$$

Note that when the thyristor conducts (T_1 is on) during $\omega t = \alpha$ to π, the thyristor current i_{T1}, the load current i_O through R_L and the source current i_s flowing through the transformer secondary winding are all one and the same.

Hence we can write

$$i_s = i_{T1} = i_o = \frac{V_O}{R} = \frac{V_m \sin \omega t}{R} \; ; for\ \alpha \le \omega t \le \pi$$

I_m is the maximum (peak) value of the load current that flows through the transformer secondary winding, through T_1 and through the load resistor R_L at the instant $\omega t = \frac{\pi}{2}$, when the input supply voltage reaches its maximum value.

When the input supply voltage decreases the load current decreases. When the supply voltage falls to zero at $\omega t = \pi$, the thyristor and the load current also falls to zero at $\omega t = \pi$. Thus the thyristor naturally turns off when the current flowing through it falls to zero at $\omega t = \pi$.

During the negative half cycle of input supply when the supply voltage reverses and becomes negative during $\omega t = \pi$ to 2π radians, the anode of thyristor is at a negative potential with respect to its cathode and as a result the thyristor is reverse biased and hence it remains cut-off (in the reverse blocking mode). The thyristor cannot conduct during its reverse biased state between $\omega t = \pi$ to 2π. An ideal thyristor under reverse biased condition behaves as an open switch and hence the load current and load voltage are zero during $\omega t = \pi$ to 2π. The maximum or peak reverse voltage that appears across the thyristor anode and cathode terminals is V_m.

The trigger angle α (delay angle or the phase angle α) is measured from the beginning of each positive half cycle to the time instant when the gate trigger pulse is applied. The thyristor conduction angle is from α to π, hence the conduction angle $\delta = (\pi - \alpha)$. The maximum conduction angle is π radians (180^0) when the trigger angle $\alpha = 0$.

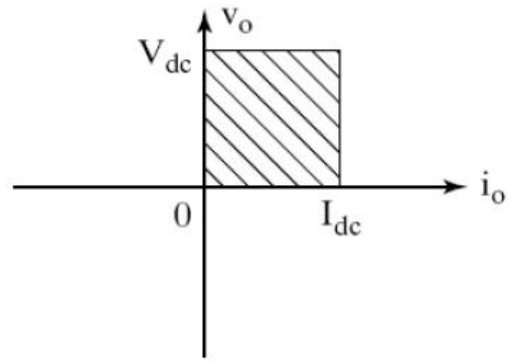


Fig.2 Quadrant Diagram

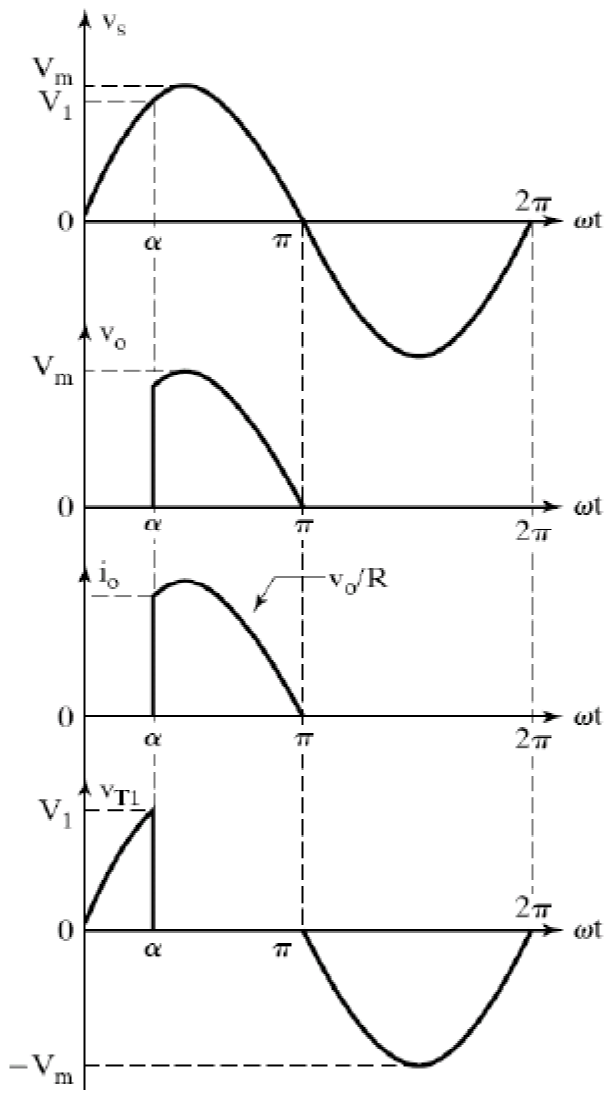


Fig.3 Waveforms of single phase half-wave controlled rectifier with resistive load

The waveforms shows the input ac supply voltage across the secondary winding of the transformer which is represented as V_s , the output voltage across the load, the output (load) current, and the thyristor voltage waveform that appears across the anode and cathode terminals.

EQUATIONS

$V_s = V_m \sin \omega t =$ the ac supply voltage across the transformer secondary.

$V_m =$ max. (peak) value of input ac supply voltage across transformer secondary.

$V_s = \frac{V_m}{\sqrt{2}}$ = RMS value of input ac supply voltage across transformer secondary.

$V_O = V_L =$ the output voltage across the load

$i_O = i_L =$ output (load) current.

When the thyristor is triggered at $\omega t = \alpha$ (an ideal thyristor behaves as a closed switch) and hence the output voltage follows the input supply voltage.

$V_O = V_L = V_m \sin \omega t$; for $\omega t = \alpha$ to π, when the thyristor is on.

$i_O = i_L = \frac{V_O}{R}$ = Load current for $\omega t = \alpha$ to π, when the thyristor is on.

TO DERIVE AN EXPRESSION FOR THE AVERAGE (DC) OUTPUT VOLTAGE ACROSS THE LOAD

If V_m is the peak input supply voltage, the average output voltage V_{dc} can be found from

$$V_{O(dc)} = \frac{1}{2\pi} \int_{\alpha}^{\pi} V_m \sin \omega t \,.\, d(\omega t)$$

$$V_{O(dc)} = \frac{V_m}{2\pi} \int_{\alpha}^{\pi} \sin \omega t \,.\, d(\omega t)$$

$V_{O(dc)} = \frac{V_m}{2\pi} [-\cos \pi + \cos \alpha]$; $\cos \pi = -1$

$$V_{O(dc)} = \frac{V_m}{2\pi} [1 + \cos \alpha]$$

$$V_m = \sqrt{2} V_s$$

The maximum average (dc) output voltage is obtained when $\alpha = 0$ and the maximum dc output voltage $V_{dc(max)} = V_{dm} = \frac{V_m}{\pi}$.

The average dc output voltage can be varied by varying the trigger angle α from 0 to a maximum of 180^0 (π radians).

We can plot the control characteristic, which is a plot of dc output voltage versus the trigger angle α by using the equation for $V_{O(dc)}$

CONTROL CHARACTERISTIC OF SINGLE PHASE HALF WAVE PHASE CONTROLLED RECTIFIER WITH RESISTIVE LOAD

The average dc output voltage is given by the expression

$$V_{O(dc)} = \frac{V_m}{2\pi}[1 + \cos\alpha]$$

We can obtain the control characteristic by plotting the expression for the dc output voltage as a function of trigger angle α

Trigger angle α in degrees	$V_{O(dc)}$	%
0	$V_{dm} = \frac{V_m}{\pi}$	100% V_{dm}
30^0	0.933 V_{dm}	93.3 % V_{dm}
60^0	0.75 V_{dm}	75 % V_{dm}
90^0	0.5 V_{dm}	50 % V_{dm}
120^0	0.25 V_{dm}	25 % V_{dm}
150^0	0.06698 V_{dm}	6.69 % V_{dm}
180^0	0	0

$$V_{dm} = \frac{V_m}{\pi} = V_{dc(max)}$$

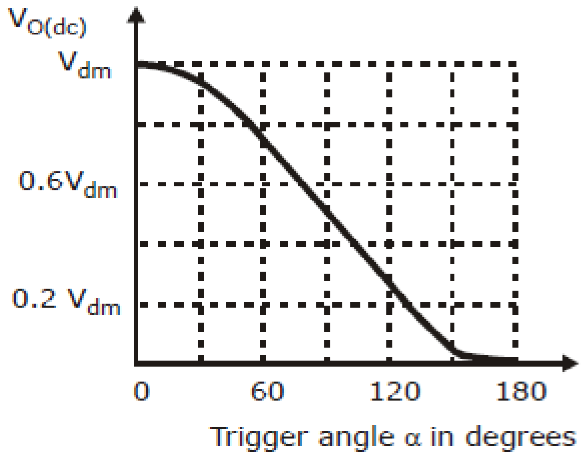


Fig.4 Control characteristic

Normalizing the dc output voltage with respect to V_{dm}, the normalized output voltage

$$V_{dm} = \frac{V_{O(dc)}}{V_{dc(max)}} = \frac{V_{dc}}{V_{dm}}$$

$$V_{dcn} = V_n = \frac{V_{dc}}{V_{dm}} = \frac{\frac{V_m}{2\pi}[1+\cos\alpha]}{\frac{V_m}{\pi}}$$

$$V_n = \frac{V_{dc}}{V_{dm}} = \frac{1}{2}(1+\cos\alpha) = V_{dcn}$$

TO DERIVE AN EXPRESSION FOR THE RMS VALUE OF OUTPUT VOLTAGE OF A SINGLE PHASE HALF WAVE CONTROLLED RECTIFIER WITH RESISTIVE LOAD

The rms output voltage is given by

$$V_{O(RMS)} = \left[\frac{1}{2\pi}\int_0^{2\pi} v_O^2.d(\omega t)\right]$$

Output voltage $v_O = V_m \sin\omega t$; for $\omega t = \alpha$ to π

$$V_{O(RMS)} = \left[\frac{1}{2\pi}\int_\alpha^{\pi} V_m^2 \sin^2\omega t.d(\omega t)\right]^{\frac{1}{2}}$$

By substituting $\sin^2 \omega t = \frac{1-\cos 2\omega t}{2}$, we get

$$V_{O(RMS)} = \left[\frac{1}{2\pi}\int_{\alpha}^{\pi} V_m^2 \frac{(1-\cos 2\omega t)}{2}.d(\omega t)\right]^{\frac{1}{2}}$$

$$V_{O(RMS)} = \left[\frac{V_m^2}{4\pi}\int_{\alpha}^{\pi} (1-\cos 2\omega t).d(\omega t)\right]^{\frac{1}{2}}$$

$$V_{O(RMS)} = \left[\frac{V_m^2}{4\pi}\left\{\int_{\alpha}^{\pi} d(\omega t) - \int_{\alpha}^{\pi} \cos 2\omega t.d(\omega t)\right\}\right]^{\frac{1}{2}}$$

$$V_{O(RMS)} = \frac{V_m}{2}\left[\frac{1}{\pi}\left\{(\omega t)\Big/_{\alpha}^{\pi} - \left(\frac{\sin 2\omega t}{2}\right)\Big/_{\alpha}^{\pi}\right\}\right]^{\frac{1}{2}}$$

$$V_{O(RMS)} = \frac{V_m}{2}\left[\frac{1}{\pi}\left((\pi-\alpha) - \frac{(\sin 2\pi - \sin 2\alpha)}{2}\right)\right]^{\frac{1}{2}} \; ; \; \sin 2\pi = 0$$

Hence we get,

$$V_{O(RMS)} = \frac{V_m}{2}\left[\frac{1}{\pi}\left((\pi-\alpha) + \frac{\sin 2\alpha}{2}\right)\right]^{\frac{1}{2}}$$

$$V_{O(RMS)} = \frac{V_m}{2\sqrt{\pi}}\left((\pi-\alpha) + \frac{\sin 2\alpha}{2}\right)^{\frac{1}{2}}$$

PERFORMANCE PARAMETERS OF PHASE CONTROLLED RECTIFIERS

Output dc power (average or dc output power delivered to the load)

$$P_{O(dc)} = V_{O(dc)} \times I_{O(dc)} \; ; \text{ i.e., } P_{dc} = V_{dc} \times I_{dc}$$

Where

$V_{O(dc)} = V_{dc} =$ average or dc value of output (load) voltage.

$I_{O(dc)} = I_{dc} =$ average or dc value of output (load) current.

Output ac power

$P_{O(ac)} = V_{O(rms)} * I_{O(rms)}$

Efficiency of Rectification (Rectification Ratio)

$$\text{Efficiency } \eta = \frac{P_{O(dc)}}{P_{O(ac)}} \quad ; \qquad \% \text{ Efficiency } \eta = \frac{P_{O(dc)}}{P_{O(ac)}} \times 100$$

The output voltage can be considered as being composed of two components

- The dc component $V_{O(dc)}$= DC or average value of output voltage.
- The ac component or the ripple components $V_{ac}=V_{rms}$=RMS value ofall the ac ripple components.

The total RMS value of output voltage is given by

$$V_{O(RMS)} = \sqrt{V_{O(dc)}^2 + V_{r(rms)}^2}$$

Therefore

$$V_{ac} = V_{r(rms)} = \sqrt{V_{O(RMS)}^2 - V_{O(dc)}^2}$$

The Ripple Factor (RF) which is a measure of the ac ripple content in the output voltage waveform. The output voltage ripple factor defined for the output voltage waveform is given by

$$r_v = RF = \frac{V_{r(rms)}}{V_{O(dc)}} = \frac{V_{ac}}{V_{dc}}$$

$$r_v = \frac{\sqrt{V_{O(RMS)}^2 - V_{O(dc)}^2}}{V_{O(dc)}} = \sqrt{\left[\frac{V_{O(RMS)}}{V_{O(dc)}}\right]^2 - 1}$$

Therefore

$$r_v = \sqrt{FF^2 - 1}$$

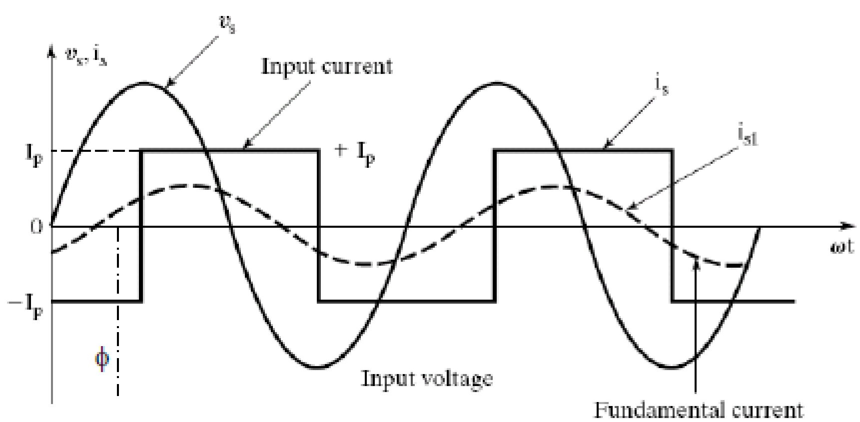
Transformer Utilization Factor (TUF)

$$TUF = \frac{P_{O(dc)}}{V_S \times I_S}$$

Where

$V_S =$ RMS value of transformer secondary output voltage (RMS supply voltage at the secondary)

$I_S =$ RMS value of transformer secondary current (RMS line or supply current).



$v_S =$ Supply voltage at the transformer secondary side.
$i_S =$ Input supply current (transformer secondary winding current).
$i_{S1} =$ Fundamental component of the input supply current.
$I_P =$ Peak value of the input supply current.
$\phi =$ Phase angle difference between (sine wave components) the fundamental components of input supply current and the input supply voltage.
$\phi =$ Displacement angle (phase angle)

For an RL load $\phi =$ Displacement angle = Load impedance angle

$$\therefore \quad \phi = \tan^{-1}\left(\frac{\omega L}{R}\right) \text{ for an RL load}$$

Displacement Factor (DF) *or* ***Fundamental Power Factor***

$$DF = \cos \phi$$

Harmonic Factor (HF) or Total Harmonic Distortion Factor (THD)

The harmonic factor is a measure of the distortion in the output waveform and is also referred to as the total harmonic distortion (THD)

Where

$$I_S = RMS\ value\ of\ input\ supply\ current$$

I_{S1} = RMS value of fundamental component of the input supply current.

Input Power Factor (PF)

$$PF = \frac{V_S I_{S1}}{V_S I_S}\cos\emptyset = \frac{I_{S1}}{I_S}\cos\emptyset$$

The Crest Factor (CF)

$$CF = \frac{I_{S(peak)}}{I_S} = \frac{\text{Peak input supply current}}{\text{RMS input supply current}}$$

For an Ideal Controlled Rectifier

FF =1 ; which means that $V_{O(RMS)} = V_{O(dc)}$

Efficiency $\eta = 100\%$;

which means that $P_{O(dc)} = P_{O(ac)}$

$$V_{ac} = V_{r(rms)} = 0$$

So that $RF = r_v = 0$

Ripple factor = 0 (ripple free converter).

TUF =1 ; which means that $P_{O(dc)}=V_S I_S$

$HF = THD = 0$; which means that $I_S = I_{S1}$

$PF = DPF$ =1 ; which means that $\phi = 0$

SINGLE PHASE HALF WAVE CONTROLLED RECTIFIER WITH AN RL LOAD

In this section we will discuss the operation and performance of a single phase half wave controlled rectifier with RL load. In practice most of the loads are of RL type. For example if we consider a single phase controlled rectifier controlling the speed of a dc motor, the load which is the dc motor winding is an RL type of load, where R represents the motor winding resistance and L represents the motor winding inductance.

A single phase half wave controlled rectifier circuit with an RL load using a thyristor T_1 (T_1 is an SCR) is shown in the figure below.

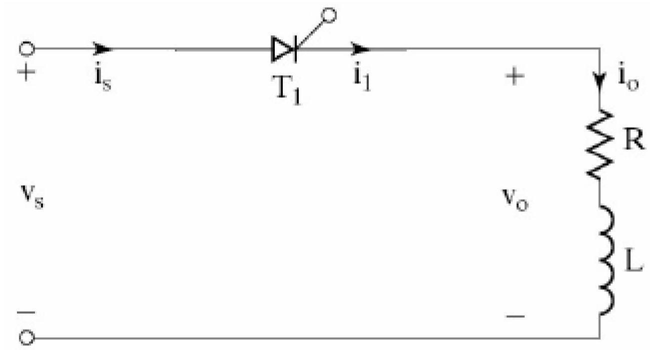
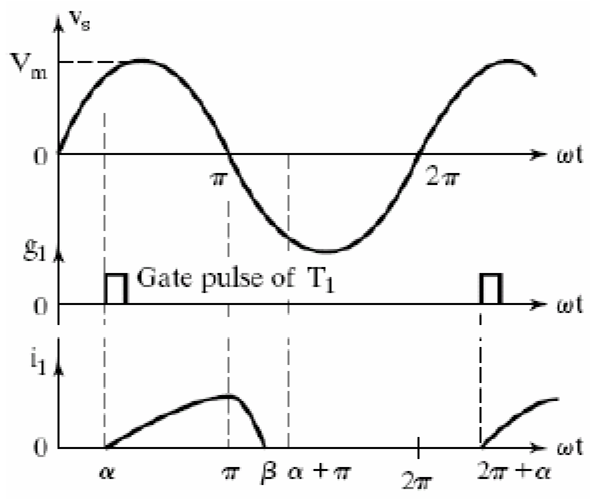


Fig.5 Circuit Diagram

The thyristor T_1 is forward biased during the positive half cycle of input supply. Let us assume that T_1 is triggered at $\omega t = \alpha$, by applying a suitable gate trigger pulse to T_1 during the positive half cycle of input supply. The output voltage across the load follows the input supply voltage when T_1 is ON. The load current i_o flows through the thyristor T_1 and through the load in the downward direction. This load current pulse flowing through T_1 can be considered as the positive current pulse. Due to the inductance in the load, the load current i_o flowing through T_1 would not fall to zero at $\omega t = \pi$, when the input supply voltage starts to become negative. A phase shift appears between the load voltage and the load current waveforms, due to the load inductance.

The thyristor T_1 will continue to conduct the load current until all the inductive energy stored in the load inductor L is completely utilized and the load current through T_1 falls to zero at $\omega t = \beta$, where β is referred to as the Extinction angle, (the value of ωt) at which the load current falls to zero. The extinction angle β is measured from the point of the beginning of the positive half cycle of input supply to the point where the load current falls to zero.

The thyristor T_1 conducts from $\omega t = \alpha$ to β. The conduction angle of T_1 is $\delta = (\beta - \alpha)$, which depends on the delay angle α and the load impedance angle ϕ. The waveforms of the input supply voltage, the gate trigger pulse of T_1, the thyristor current, the load current and the load voltage waveforms appear as shown in the figure below.



$i_1 = i_O = i_S$

Fig.6 Input supply voltage & Thyristor current waveforms

β is the extinction angle which depends upon the load inductance value.

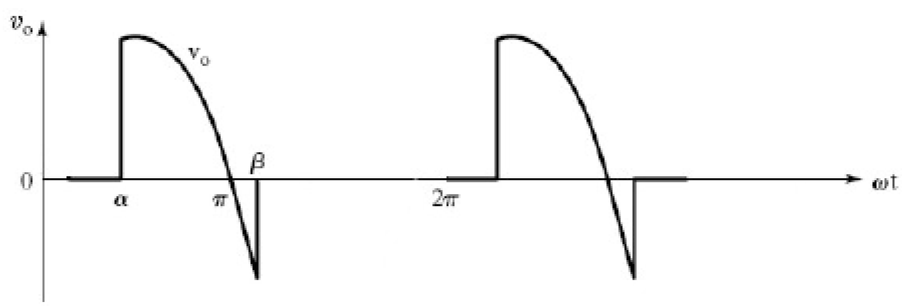


Fig.7 Output (load) voltage waveform of a single phase half wave controlled rectifier with RL load

From β to 2π, the thyristor remains cut-off as it is reverse biased and behaves as an open switch. The thyristor current and the load current are zero and the output voltage also remains at zero during the non conduction time interval between β to 2π. In the next cycle the thyristor is triggered again at a phase angle of $(2\pi+\alpha)$, and the same operation repeats.

TO DERIVE AN EXPRESSION FOR THE OUTPUT (INDUCTIVE LOAD) CURRENT, DURING $\omega t = \alpha$ to β WHEN THYRISTOR T_1 CONDUCTS

Considering sinusoidal input supply voltage we can write the expression for the supply voltage as

$V_s = V_m \sin\omega t =$ instantaneous value of the input supply voltage.

Let us assume that the thyristor T_1 is triggered by applying the gating signal to T_1 at $\omega t = \alpha$. The load current which flows through the thyristor T_1 during $\omega t = \alpha$ to β can be found from the equation

$$L\left(\frac{di_o}{dt}\right) + Ri_o = V_m \sin\omega t$$

The solution of the above differential equation gives the general expression for the output load current which is of the form

$$i_o = \frac{V_m}{Z}\sin(\omega t - \phi) + A_1 e^{\frac{-t}{\tau}}$$

Where $V_m = \sqrt{2}V_S =$ maximum or peak value of input supply voltage

$$Z = \sqrt{R^2 + (\omega L)^2}$$

$\phi = \tan^{-1}\frac{\omega L}{R}$ = Load impedance angle (power factor angle of load).

$\tau = \frac{L}{R}$ = Load circuit time constant.

Therefore the general expression for the output load current is given by the equation

$$i_o = \frac{V_m}{Z}\sin(\omega t - \phi) + A_1 e^{\frac{-Rt}{L}}$$

The value of the constant A_1 can be determined from the initial condition. i.e. initial value of load current $i_o = 0$, at $\omega t = \alpha$. Hence from the equation for i_o equating i_o to zero and substituting $\omega t = \alpha$, we get

$$i_o = 0 = \frac{V_m}{Z}\sin(\alpha - \phi) + A_1 e^{\frac{-Rt}{L}}$$

Therefore

$$A_1 e^{\frac{-R}{L}t} = \frac{-V_m}{Z}\sin(\alpha - \phi)$$

$$A_1 = \frac{1}{e^{\frac{-R}{L}t}}\left[\frac{-V_m}{Z}\sin(\alpha - \phi)\right]$$

$$A_1 = e^{\frac{+R}{L}t}\left[\frac{-V_m}{Z}\sin(\alpha - \phi)\right]$$

$$A_1 = e^{\frac{R(\alpha t)}{\omega L}} \left[\frac{-V_m}{Z} \sin(\alpha - \phi) \right]$$

By substituting $\omega t = \alpha$, we get the value of constant A_1as

$$\boldsymbol{A_1 = e^{\frac{R(\alpha)}{\omega L}} \left[\frac{-V_m}{Z} \sin(\alpha - \phi) \right]}$$

Substituting the value of constant $A_1$from the above equation into the expression for i_o, we obtain

$$i_O = \frac{V_m}{Z} \sin(\omega t - \phi) + e^{\frac{-R}{L}t} e^{\frac{R(\alpha)}{\omega L}} \left[\frac{-V_m}{Z} \sin(\alpha - \phi) \right] \; ;$$

$$i_O = \frac{V_m}{Z} \sin(\omega t - \phi) + e^{\frac{-R(\omega t)}{\omega L}} e^{\frac{R(\alpha)}{\omega L}} \left[\frac{-V_m}{Z} \sin(\alpha - \phi) \right]$$

$$i_O = \frac{V_m}{Z} \sin(\omega t - \phi) + e^{\frac{-R}{\omega L}(\omega t - \alpha)} \left[\frac{-V_m}{Z} \sin(\alpha - \phi) \right]$$

Therefore we obtain the final expression for the inductive load current of a single phase half wave controlled rectifier with RL load as

$$i_O = \frac{V_m}{Z} \left[\sin(\omega t - \phi) - \sin(\alpha - \phi) e^{\frac{-R}{\omega L}(\omega t - \alpha)} \right] \; ; \quad \text{Where } \alpha \le \omega t \le \beta .$$

The above expression also represents the thyristor current i_{T1}, during the conduction time interval of thyristor $T_1$from $\omega t = \alpha$ to β .

TO CALCULATE EXTINCTION ANGLE β

The extinction angle β, which is the value of ωt at which the load current $i_o$falls to zero and $T_1$is turned off can be estimated by using the condition that $i_o = 0$, at $\omega t = \beta$.

By using the above expression for the output load current, we can write

$$i_o = 0 = \frac{V_m}{Z}\left[\sin(\beta-\phi) - \sin(\alpha-\phi)e^{\frac{-R}{\omega L}(\beta-\alpha)}\right]$$

As $\frac{V_m}{Z} \neq 0$, we can write

$$\left[\sin(\beta-\phi) - \sin(\alpha-\phi)e^{\frac{-R}{\omega L}(\beta-\alpha)}\right] = 0$$

Therefore we obtain the expression

$$\sin(\beta - \phi) = \sin(\alpha - \phi)e^{\frac{-R}{\omega L}(\beta-\alpha)}$$

The extinction angle β can be determined from this transcendental equation by using the iterative method of solution (trial and error method). After β is calculated, we can determine the thyristor conduction angle $\delta = (\beta - \alpha)$.

β is the extinction angle which depends upon the load inductance value. Conduction angle δ increases as α is decreased for a specific value of β.

Conduction angle $\delta = (\beta - \alpha)$; for a purely resistive load or for an RL load when the load inductance L is negligible the extinction angle $\beta = \pi$ and the conduction angle $\delta = (\pi - \alpha)$

Equations

$$V_s = V_m \sin \omega t = \text{Input supply voltage}$$

$$V_o = V_L = V_m \sin \omega t = \text{Output load voltage for} \omega t = \alpha \text{ to } \beta$$

when the thyristor T_1conducts ($T_1$is on).

Expression for the load current (thyristor current): for $\omega t = \alpha$ to β

$$i_o = \frac{V_m}{Z}\left[\sin(\omega t - \phi) - \sin(\alpha - \phi)e^{\frac{-R}{\omega L}(\omega t-\alpha)}\right]$$

Extinction angle β can be calculated using the equation

$$\sin(\omega t - \phi) - \sin(\alpha - \phi) e^{\frac{-R}{\omega L}(\beta - \alpha)}$$

TO DERIVE AN EXPRESSION FOR AVERAGE (DC) LOAD VOLTAGE

$$V_{O(dc)} = V_L = \frac{1}{2\pi} \int_0^{2\pi} v_O . d(\omega t)$$

$$V_{O(dc)} = V_L = \frac{1}{2\pi} \left[\int_0^{\alpha} v_O . d(\omega t) + \int_{\alpha}^{\beta} v_O . d(\omega t) + \int_{\beta}^{2\pi} v_O . d(\omega t) \right] ;$$

$v_O = 0$ for $\omega t = 0$ to α & for $\omega t = \beta$ to 2π ;

$$V_{O(dc)} = V_L = \frac{1}{2\pi} \left[\int_{\alpha}^{\beta} v_O . d(\omega t) \right] ; v_O = V_m \sin \omega t \text{ for } \omega t = \alpha \text{ to } \beta$$

$$V_{O(dc)} = V_L = \frac{1}{2\pi} \left[\int_{\alpha}^{\beta} V_m \sin \omega t . d(\omega t) \right]$$

$$V_{O(dc)} = V_L = \frac{V_m}{2\pi} \left[-\cos \omega t \Big/_{\alpha}^{\beta} \right] = \frac{V_m}{2\pi} (\cos \alpha - \cos \beta)$$

$$V_{O(dc)} = V_L = \frac{V_m}{2\pi} (\cos \alpha - \cos \beta)$$

Note: During the period $\omega t = \pi$ to β, we can see from the output load voltage waveform that the instantaneous output voltage is negative and this reduces the average or the dc output voltage when compared to a purely resistive load.

Average DC Load Current

$$I_{O(dc)} = I_{L(Avg)} = \frac{V_{O(dc)}}{R_L} = \frac{V_m}{2\pi RL}(\cos\alpha - \cos\beta)$$

SINGLE PHASE HALF WAVE CONTROLLED RECTIFIER WITH RL LOAD AND FREE WHEELING DIODE

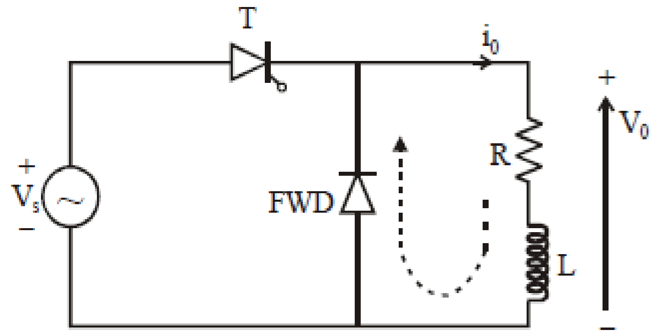


Fig.8 Single Phase Half Wave Controlled Rectifier with RL Load and Free Wheeling Diode (FWD)

With a RL load it was observed that the average output voltage reduces. This disadvantage can be overcome by connecting a diode across the load as shown in figure. The diode is called as a *Free Wheeling Diode (FWD)*. The waveforms are shown below.

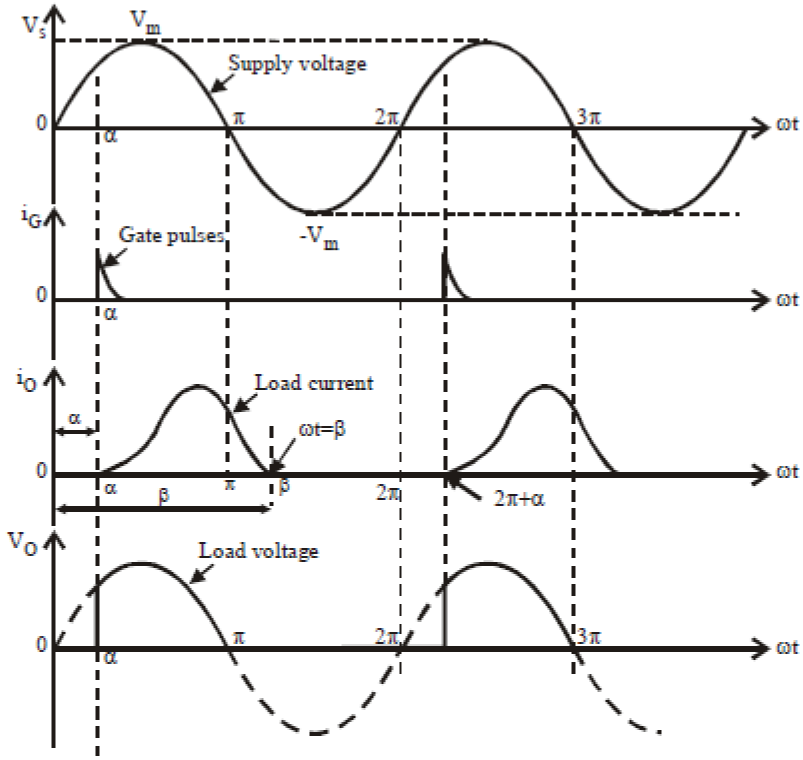


Fig.9 Wave Forms

At $\omega t = \pi$, the source voltage v_S falls to zero and as v_S becomes negative, the free wheeling diode is forward biased. The stored energy in the inductance maintains the load current flow through R, L, and the FWD. Also, as soon as the FWD is forward biased, at $\omega t = \pi$, the SCR becomes reverse biased, the current through it becomes zero and the SCR turns off. During the period $\omega t = \pi$ to β, the load current flows through FWD (free wheeling load current) and decreases exponentially towards zero at $\omega t = \beta$.

Also during this free wheeling time period the load is shorted by the conducting FWD and the load voltage is almost zero, if the forward voltage drop across the conducting FWD is neglected. Thus there is no negative region in the load voltage wave form. This improves the average output voltage.

The average output voltage $V_{dc} = \frac{V_m}{2\pi}(1 + \cos\alpha)$, which is the same as that of a purely resistive load. The output voltage across the load appears similar to the output voltage of a purely resistive load.

The following points are to be noted.

- If the inductance value is not very large, the energy stored in the inductance is able to maintain the load current only upto $\omega t = \beta$, where $\pi < \beta < 2\pi$, well before the next gate pulse and the load current tends to become discontinuous.
- During the conduction period α to π, the load current is carried by the SCR and during the free wheeling period π to β, the load current is carried by the free wheeling diode.
- The value of β depends on the value of R and L and the forward resistance of the FWD. Generally $\pi < \beta < 2\pi$.

If the value of the inductance is very large, the load current does not decrease to zero during the free wheeling time interval and the load current waveform appears as shown in the figure.

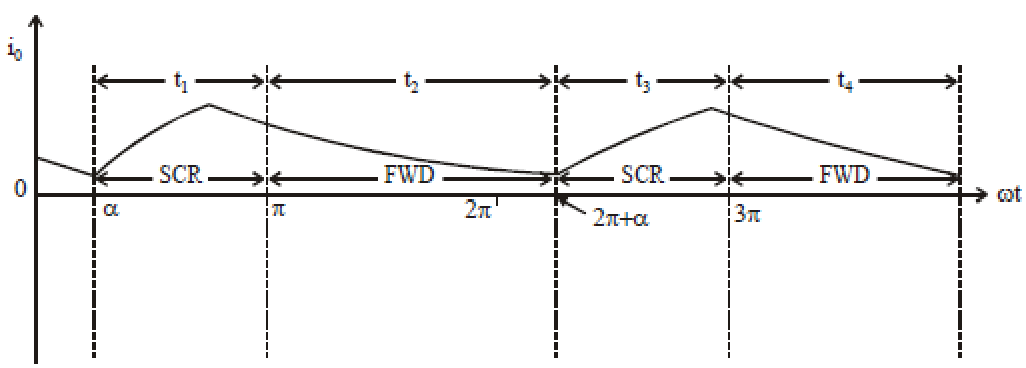


Fig.10 Waveform of Load Current in Single Phase Half Wave Controlled Rectifier with a Large Inductance and FWD

During the periods $t_1, t_3, \ldots$.. the SCR carries the load current and during the periods $t_2, t_4, \ldots$.. the FWD carries the load current.

It is to be noted that

- The load current becomes continuous and the load current does not fall to zero for large value of load inductance.

- The ripple in the load current waveform (the amount of variation in the output load current) decreases.

SINGLE PHASE HALF WAVE CONTROLLED RECTIFIER WITH A GENERAL LOAD

A general load consists of R, L and a DC source 'E' in the load circuit

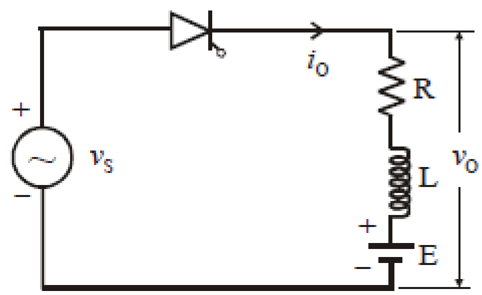


Fig.11 Circuit Diagram

In the half wave controlled rectifier circuit shown in the figure, the load circuit consists of a dc source 'E' in addition to resistance and inductance. When the thyristor is in the cut-off state, the current in the circuit is zero and the cathode will be at a voltage equal to the dc voltage in the load circuit i.e. the cathode potential will be equal to 'E'. The thyristor will be forward biased for anode supply voltage greater than the load dc voltage.

When the supply voltage is less than the dc voltage 'E' in the circuit the thyristor is reverse biased and hence the thyristor cannot conduct for supply voltage less than the load circuit dc voltage.

The value of ωt at which the supply voltage increases and becomes equal to the load circuit dc voltage can be calculated by using the equation $V_m \sin \omega t$ = E. If we assume the value of ωt is equal to γ then we can write $V_m \sin\gamma$ = E . Therefore γ is calculated as $\gamma = \sin^{-1}\left(\frac{E}{V_m}\right)$.

For trigger angle $\alpha < \gamma$, the thyristor conducts only from $\omega t = \gamma$ to β.

For trigger angle $\alpha > \gamma$, the thyristor conducts from $\omega t = \alpha$ to β

The waveforms appear as shown in the figure

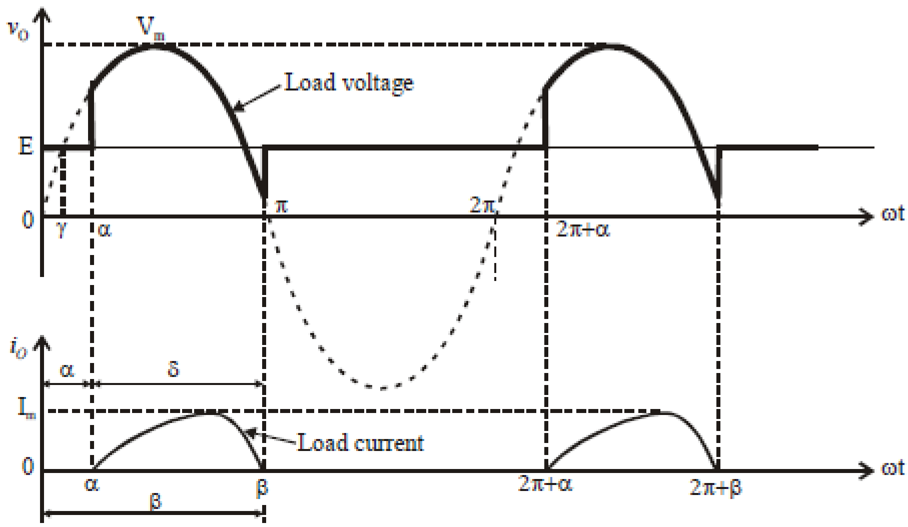


Fig.12 Wave Forms

Equations

$$V_s = V_m \sin \omega t = \text{Input supply voltage}$$

$$V_o = V_m \sin \omega t = \text{ Output load voltage for} \omega t = \alpha \text{ to } \beta$$

$V_o = E$ for $\omega t = 0$ $to\ \alpha\ and\ for\ \omega t = \beta\ to\ 2\pi$

Expression for the Load Current

When the thyristor is triggered at a delay angle of α, the equation for the circuit can be written as

$V_m \sin \omega t = i_o * R + L\left(\frac{di_o}{dt}\right) + E\ ;\ \alpha \leq \omega \leq \beta$

The general expression for the output load current can be written as

$$i_O = \frac{V_m}{Z}\sin(\omega t - \phi) - \frac{E}{R} + Ae^{\frac{-t}{\tau}}$$

Where

$$Z = \sqrt{R^2 + (\omega L)^2} = \text{Load Impedance}$$

$$\phi = \tan^{-1}\left(\frac{\omega L}{R}\right) = \text{Load impedance angle}$$

$$\tau = \frac{L}{R} = \text{Load circuit time constant}$$

The general expression for the output load current can be written as

$$i_o = \frac{V_m}{Z}\sin(\omega t - \phi) - \frac{E}{R} + Ae^{\frac{-R}{L}t}$$

To find the value of the constant 'A' apply the initial condition at $\omega t = \alpha$, load current $i_O = 0$.

Equating the general expression for the load current to zero at $\omega t = \alpha$, we get

$$i_o = 0 = \frac{V_m}{Z}\sin(\alpha - \phi) - \frac{E}{R} + Ae^{\frac{-R}{L}\frac{\alpha}{\omega}}$$

We obtain the value of constant 'A' as

$$A = \left[\frac{E}{R} - \frac{V_m}{Z}\sin(\alpha - \phi)\right]e^{\frac{R}{L}\frac{\alpha}{\omega}}$$

Substituting the value of the constant 'A' in the expression for the load current, we get the complete expression for the output load current as

$$i_o = \frac{V_m}{Z}\sin(\alpha - \phi) - \frac{E}{R} + \left[\frac{E}{R} - \frac{V_m}{Z}\sin(\alpha - \phi)\right]e^{\frac{-R}{\omega L}(\omega t - \alpha)}$$

The Extinction angle β can be calculated from the final condition that the output current $i_O = 0$ at $\omega t = \beta$. By using the above expression we get,

$$i_o = 0 = \frac{V_m}{Z}\sin(\beta - \phi) - \frac{E}{R} + \left[\frac{E}{R} - \frac{V_m}{Z}\sin(\alpha - \phi)\right]e^{\frac{-R}{\omega L}(\beta - \alpha)}$$

To derive an expression for the average or dc load voltage

$$V_{O(dc)} = \frac{1}{2\pi}\int_0^{2\pi} v_O.d(\omega t)$$

$$V_{O(dc)} = \frac{1}{2\pi}\left[\int_0^{\alpha} v_O.d(\omega t) + \int_{\alpha}^{\beta} v_O.d(\omega t) + \int_{\beta}^{2\pi} v_O.d(\omega t)\right]$$

$v_O = V_m \sin\omega t =$ Output load voltage for $\omega t = \alpha$ to β

$v_O = E$ for $\omega t = 0$ to α & for $\omega t = \beta$ to 2π

$$V_{O(dc)} = \frac{1}{2\pi}\left[\int_0^{\alpha} E.d(\omega t) + \int_{\alpha}^{\beta} V_m \sin\omega t + \int_{\beta}^{2\pi} E.d(\omega t)\right]$$

$$V_{O(dc)} = \frac{1}{2\pi}\left[E(\omega t)\Big/_0^{\alpha} + V_m(-\cos\omega t)\Big/_{\alpha}^{\beta} + E(\omega t)\Big/_{\beta}^{2\pi}\right]$$

$$V_{O(dc)} = \frac{1}{2\pi}\left[E(\alpha - 0) - V_m(\cos\beta - \cos\alpha) + E(2\pi - \beta)\right]$$

$$V_{O(dc)} = \frac{V_m}{2\pi}\left[(\cos\alpha - \cos\beta)\right] + \frac{E}{2\pi}(2\pi - \beta + \alpha)$$

$$V_{O(dc)} = \frac{V_m}{2\pi}(\cos\alpha - \cos\beta) + \left[\frac{2\pi - (\beta - \alpha)}{2\pi}\right]E$$

Conduction angle of thyristor $\delta = (\beta - \alpha)$

RMS Output Voltage can be calculated by using the expression

$$V_{O(RMS)} = \sqrt{\frac{1}{2\pi}\left[\int_0^{2\pi} v_O^2.d(\omega t)\right]}$$

DISADVANTAGES OF SINGLE PHASE HALF WAVE CONTROLLED RECTIFIERS

Single phase half wave controlled rectifier gives

- Low dc output voltage.
- Low dc output power and lower efficiency.

- Higher ripple voltage & ripple current.
- Higher ripple factor.
- Low transformer utilization factor.
- The input supply current waveform has a dc component which can result in dc
- saturation of the transformer core.

Single phase half wave controlled rectifiers are rarely used in practice as they give low dc output and low dc output power. They are only of theoretical interest. The above disadvantages of a single phase half wave controlled rectifier can be over come by using a full wave controlled rectifier circuit. Most of the practical converter circuits use full wave controlled rectifiers.

SINGLE PHASE FULL WAVE CONTROLLED RECTIFIERS

Single phase full wave controlled rectifier circuit combines two half wave controlled rectifiers in one single circuit so as to provide two pulse output across the load. Both the half cycles of the input supply are utilized and converted into a unidirectional output current through the load so as to produce a two pulse output waveform. Hence a full wave controlled rectifier circuit is also referred to as a two pulse converter.

Single phase full wave controlled rectifiers are of various types

- Single phase full wave controlled rectifier using a center tapped transformer (two pulse converter with mid point configuration).
- Single phase full wave bridge controlled rectifier

❖ Half controlled bridge converter (semi converter).
❖ Fully controlled bridge converter (full converter).

SINGLE PHASE FULL WAVE CONTROLLED RECTIFIER USING A CENTER TAPPED TRANSFORMER

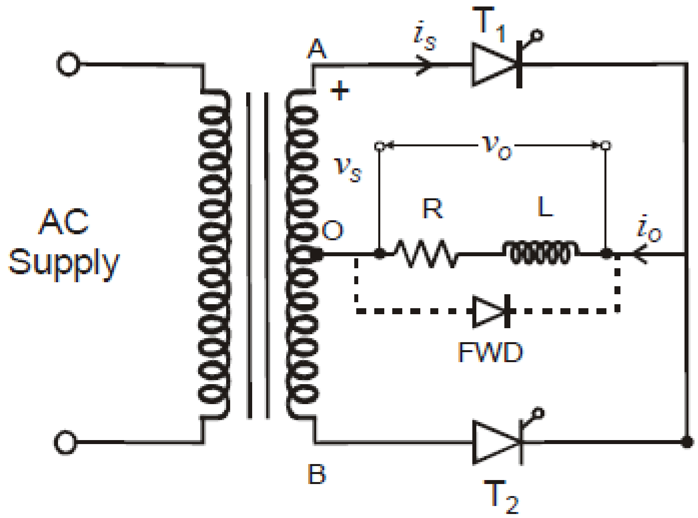


Fig.13 Circuit Diagram

$\boldsymbol{V_S}$ = Supply Voltage across the upper half of the transformer secondary winding

$$V_S = V_{AO} = V_m \sin \omega t$$

$V_{BO} = -V_{AO} = -V_m \sin \omega t$ = supply voltage across the lower half of the transformer secondary winding.

This type of full wave controlled rectifier requires a center tapped transformer and two thyristors $T_1$and T_2. The input supply is fed through the mains supply transformer, the primary side of the transformer is connected to the ac line voltage which is available (normally the primary supply voltage is 230V RMS ac supply voltage at 50Hz supply frequency in India). The secondary side of the transformer has three lines and the center point of the transformer (center line) is used as the reference point to measure the input and output voltages.

The upper half of the secondary winding and the thyristor T_1along with the load act as a half wave controlled rectifier, the lower half of the secondary winding and the thyristor $T_2$with the common load act as the second half wave controlled rectifier so as to produce a full wave load voltage waveform.

There are two types of operations possible.

- Discontinuous load current operation, which occurs for a purely resistive load or an RL load with low inductance value.

- Continuous load current operation which occurs for an RL type of load with large load inductance.

Discontinuous Load Current Operation (for low value of load inductance)

Generally the load current is discontinuous when the load is purely resistive or when the RL load has a low value of inductance.

During the positive half cycle of input supply, when the upper line of the secondary winding is at a positive potential with respect to the center point 'O' the thyristor T_1 is forward biased and it is triggered at a delay angle of α. The load current flows through the thyristor T_1, through the load and through the upper part of the secondary winding, during the period α to β, when the thyristor T_1 conducts.

The output voltage across the load follows the input supply voltage that appears across the upper part of the secondary winding from $\omega t = \alpha$ to β. The load current through the thyristor T_1 decreases and drops to zero at $\omega t = \beta$, where $\beta > \pi$ for RL type of load and the thyristor T_1 naturally turns off at $\omega t = \beta$.

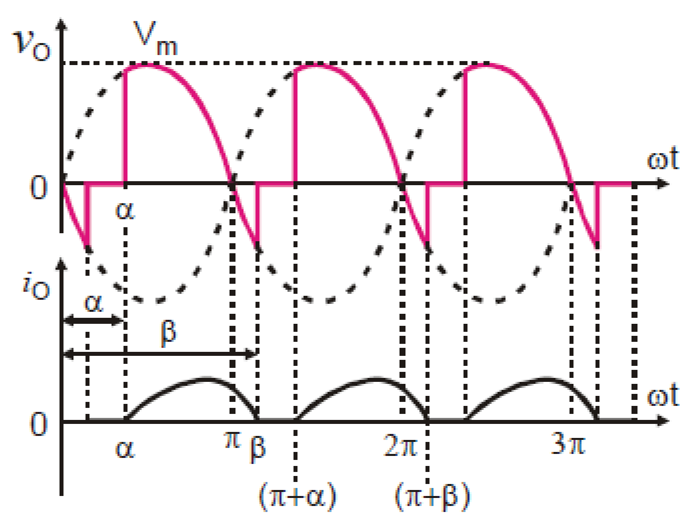


Fig.14 Waveform for Discontinuous Load Current Operation without FWD

During the negative half cycle of the input supply the voltage at the supply line 'A' becomes negative whereas the voltage at line 'B' (at the lower side of the secondary winding) becomes positive with respect to the center point 'O'. The thyristor $T_2$is forward biased during the negative half cycle and it is triggered at a delay angle of $(\pi + \alpha)$. The current flows through the thyristor T_2, through the load, and through the lower part of the secondary winding when T_2conducts during the negative half cycle the load is connected to the lower half of the secondary winding when T_2conducts.

For purely resistive loads when L = 0, the extinction angle $\beta = \pi$. The load current falls to zero at $\omega t = \beta = \pi$, when the input supply voltage falls to zero at $\omega t = \pi$. The load current and the load voltage waveforms are in phase and there is no phase shift between the load voltage and the load current waveform in the case of a purely resistive load.

For low values of load inductance the load current would be discontinuous and the extinction angle $\beta > \pi$ but $\beta < (\pi + \alpha)$

For large values of load inductance the load current would be continuous and does not fall to zero. The thyristor T_1conducts from α to $(\pi + \alpha)$, until the next thyristor T_2 is triggered.

When T_2 is triggered at $\omega t = (\pi + \alpha)$, the thyristor T_1will be reverse biased and hence T_1turns off.

TO DERIVE AN EXPRESSION FOR THE DC OUTPUT VOLTAGE OF A SINGLE PHASE FULL WAVE CONTROLLED RECTIFIER WITH RL LOAD (WITHOUT FREE WHEELING DIODE (FWD))

The average or dc output voltage of a full-wave controlled rectifier can be calculated by finding the average value of the output voltage waveform over one output cycle (i.e., π radians) and note that the output pulse repetition time is $\frac{T}{2}$ seconds where T represents the input supply time period and $T = \frac{1}{f}$; where f = input supply frequency.

Assuming the load inductance to be small so that $\beta > \pi$, $\beta < (\pi + \alpha)$ we obtain discontinuous load current operation. The load current flows through T_1form $\omega t = \alpha$ to β, where α is the trigger angle of thyristor $T_1$and β is the extinction angle where the load current through T_1falls to zero at $\omega t = \beta$. Therefore the average or dc output voltage can be obtained by using the expression.

$$V_{O(dc)} = V_{dc} = \frac{2}{2\pi} \int_{\omega t=\alpha}^{\beta} v_O . d(\omega t)$$

$$V_{O(dc)} = V_{dc} = \frac{1}{\pi} \int_{\omega t=\alpha}^{\beta} v_O . d(\omega t)$$

$$V_{O(dc)} = V_{dc} = \frac{1}{\pi}\left[\int_{\alpha}^{\beta} V_m \sin \omega t . d(\omega t)\right]$$

$$V_{O(dc)} = V_{dc} = \frac{V_m}{\pi}\left[-\cos \omega t \Big/_{\alpha}^{\beta}\right]$$

$$V_{O(dc)} = V_{dc} = \frac{V_m}{\pi}(\cos\alpha - \cos\beta)$$

Therefore $V_{O(dc)} = \frac{V_m}{\pi}(\cos\alpha - \cos\beta)$, for discontinuous load current operation,

$\pi < \beta < (\pi + \alpha$ **)**

When the load inductance is small and negligible that is $L \approx 0$, the extinction angle $\beta = \pi$ radians . Hence the average or dc output voltage for resistive load is obtained as

$\boldsymbol{V_{O(dc)} = \frac{V_m}{\pi}(\cos\alpha - \cos\pi)\ ;\ \cos\pi = -1}$

$$V_{O(dc)} = \frac{V_m}{\pi}(\cos\alpha - (-1))$$

$$V_{O(dc)} = \frac{V_m}{\pi}(1 + \cos\alpha)\ ;\ \text{for resistive load, when } L \approx 0$$

THE EFFECT OF LOAD INDUCTANCE

Due to the presence of load inductance the output voltage reverses and becomes negative during the time period $\omega t = \pi$ to β. This reduces the dc output voltage. To prevent this reduction of dc output voltage due to the negative region in the output load voltage waveform, we can connect a free wheeling diode across the load. The output voltage waveform and the dc output voltage obtained would be the same as that for a full wave controlled rectifier with resistive load.

When the Free wheeling diode (FWD) is connected across the load

When $T_1$is triggered at $\omega t = \alpha$, during the positive half cycle of the input supply the FWD is reverse biased during the time period $\omega t = \alpha$ to π. FWD remains reverse biased and cut-off from $\omega t = \alpha$ to π. The load current flows through the conducting thyristor T_1, through the RL load and through upper half of the transformer secondary winding during the time period α to π.

At $\omega t = \pi$, when the input supply voltage across the upper half of the secondary winding reverses and becomes negative the FWD turns-on. The load current continues to flow through the FWD from $\omega t = \pi$ to β.

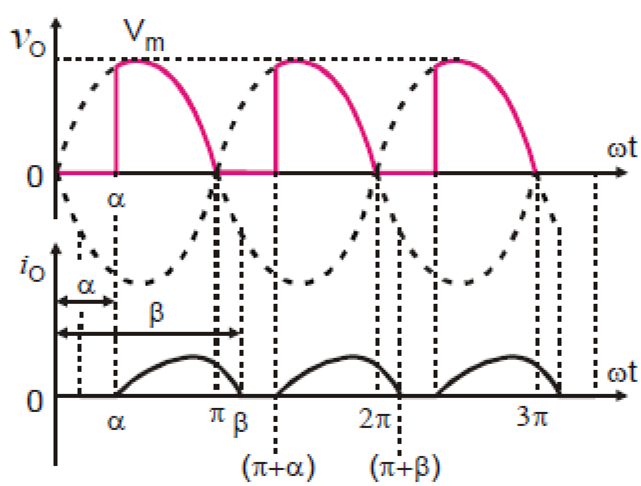


Fig.15 Waveform for Discontinuous Load Current Operation with FWD

EXPRESSION FOR THE DC OUTPUT VOLTAGE OF A SINGLE PHASE FULL WAVE CONTROLLED RECTIFIER WITH RL LOAD AND FWD

$$V_{O(dc)} = V_{dc} = \frac{1}{\pi} \int_{\omega t=0}^{\pi} V_O . d\omega t$$

Thyristor $T_1$is triggered at $\omega t = \alpha$. T_1conducts from $\omega t = \alpha$ to π.

Output voltage $V_O = V_m \sin \omega t$; for $\omega t = \alpha \; to \; \pi$.

FWD conducts from $\omega t = \pi$ to β and 0 $v_O \approx$during discontinuous load current

Therefore $$V_{O(dc)} = V_{dc} = \frac{1}{\pi} \int_{\alpha}^{\pi} V_m \sin \omega t . d(\omega t)$$

$$V_{O(dc)} = V_{dc} = \frac{V_m}{\pi} \left[-\cos \omega t \Big/_{\alpha}^{\pi} \right]$$

$$V_{O(dc)} = V_{dc} = \frac{V_m}{\pi} [-\cos \pi + \cos \alpha] \quad ; \quad \cos \pi = -1$$

Therefore $$V_{O(dc)} = V_{dc} = \frac{V_m}{\pi} (1 + \cos \alpha)$$

The DC output voltage $V_{dc}$is same as the DC output voltage of a single phase full wave controlled rectifier with resistive load. Note that the dc output voltage of a single phase full wave controlled rectifier is two times the dc output voltage of a half wave controlled rectifier.

CONTROL CHARACTERISTICS OF A SINGLE PHASE FULL WAVE CONTROLLED RECTIFIER WITH R LOAD OR RL LOAD WITH FWD

The control characteristic can be obtained by plotting the dc output voltage V_{dc}versus the trigger angle α.

The average or dc output voltage of a single phase full wave controlled rectifier circuit with R load or RL load with FWD is calculated by using the equation

$$V_{O(dc)} = V_{dc} = \frac{V_m}{\pi}(1 + \cos\alpha)$$

V_{dc}can be varied by varying the trigger angle α from 0 to 180^0. (i.e., therange of trigger angle α is from 0 to π radians).

Maximum dc output voltage is obtained when $\alpha = 0$

$$V_{dc(\max)} = V_{dc} = \frac{V_m}{\pi}(1+\cos 0) = \frac{2V_m}{\pi}$$

$V_{dc(\max)} = V_{dc} = \frac{2V_m}{\pi}$ for a single phase full wave controlled rectifier.

Normalizing the dc output voltage with respect to its maximum value, we can write the normalized dc output voltage as

$$V_{dcn} = V_n = \frac{V_{dc}}{V_{dc(\max)}} = \frac{V_{dc}}{V_{dm}}$$

$$V_{dcn} = V_n = \frac{\frac{V_m}{\pi}(1+\cos\alpha)}{\left(\frac{2V_m}{\pi}\right)} = \frac{1}{2}(1+\cos\alpha)$$

$$V_{dcn} = V_n = \frac{1}{2}(1+\cos\alpha) = \frac{V_{dc}}{V_{dm}}$$

$$V_{dc} = \frac{1}{2}(1+\cos\alpha)V_{dm}$$

Trigger angle α in degrees	$V_{O(dc)}$	**Normalized dc output voltage V_n**
0	$V_{dm} = \frac{2V_m}{\pi} = 0.636619V_m$	1
30^0	$0.593974\ V_m$	0.9330
60^0	$0.47746\ V_m$	0.75
90^0	$0.3183098\ V_m$	0.5
120^0	$0.191549\ V_m$	0.25
150^0	$0.04264\ V_m$	0.06698
180^0	0	0

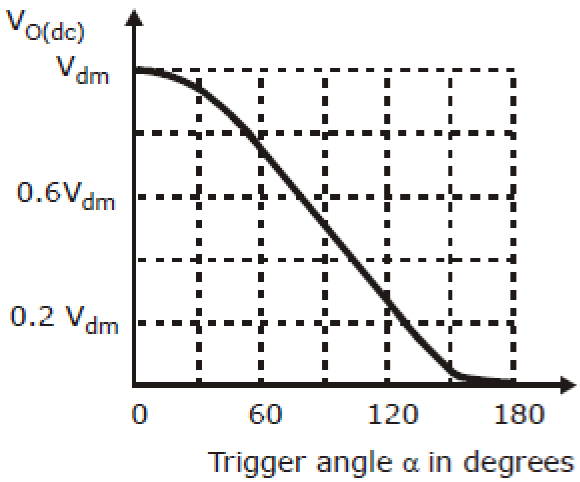


Fig.16 Control characteristic of a single phase full wave controlled rectifier with R load or RL load with FWD

CONTINUOUS LOAD CURRENT OPERATION (WITHOUT FWD)

For large values of load inductance the load current flows continuously without decreasing and falling to zero and there is always a load current flowing at any point of time. This type of operation is referred to as continuous current operation.

Generally the load current is continuous for large load inductance and for low trigger angles.

The load current is discontinuous for low values of load inductance and for large values of trigger angles.

The waveforms for continuous current operation are as shown.

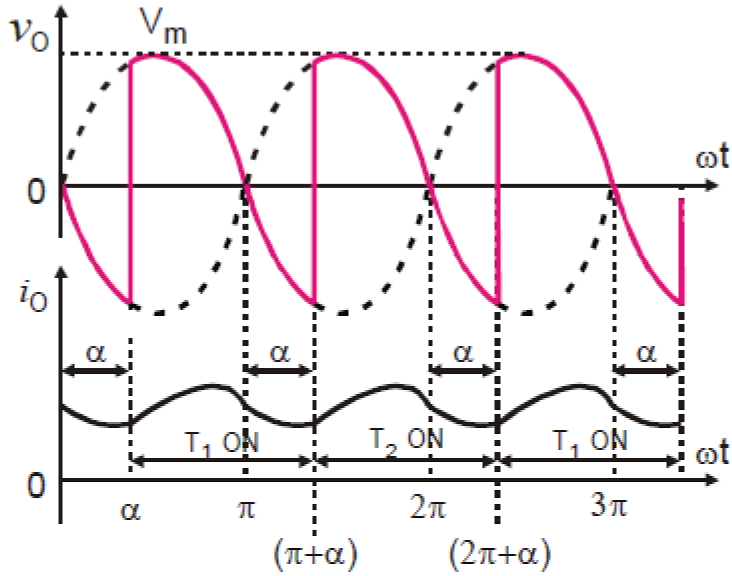


Fig.17 Load voltage and load current waveform of a single phase full wave controlled rectifier with RL load & without FWD for continuous load current operation

In the case of continuous current operation the thyristor T_1which is triggered at a delay angle of α, conducts from $\omega t = \alpha$ to $(\pi + \alpha)$. Output voltage follows the input supply voltage across the upper half of the transformer secondary winding $v_O = v_{AO} = V_m \sin \omega t$.

The next thyristor $T_2$is triggered at $\omega t = (\pi + \alpha)$, during the negative half cycle input supply. As soon as $T_2$is triggered at $\omega t = (\pi + \alpha)$, the thyristor T_1will be reverse biased and T_1turns off due to natural commutation (ac line commutation). The load current flows through the thyristor $T_2$from $\omega t = (\pi + \alpha)$ to $(2\pi + \alpha)$.

Output voltage across the load follows the input supply voltage across the lower half of the transformer secondary winding $v_O = v_{BO} = -V_m \sin \omega t$.

Each thyristor conducts for π radians (180^0) in the case of continuous current operation.

TO DERIVE AN EXPRESSION FOR THE AVERAGE OR DC OUTPUT VOLTAGE OF SINGLE PHASE FULL WAVE CONTROLLED RECTIFIER WITH LARGE LOAD INDUCTANCE ASSUMING CONTINUOUS LOAD CURRENT OPERATION.

$$V_{O(dc)} = V_{dc} = \frac{1}{\pi}\int_{\omega t=\alpha}^{(\pi+\alpha)} v_O . d(\omega t)$$

$$V_{O(dc)} = V_{dc} = \frac{1}{\pi}\left[\int_{\alpha}^{(\pi+\alpha)} V_m \sin\omega t . d(\omega t)\right]$$

$$V_{O(dc)} = V_{dc} = \frac{V_m}{\pi}\left[-\cos\omega t \Big/_{\alpha}^{(\pi+\alpha)}\right]$$

$$V_{O(dc)} = V_{dc} = \frac{V_m}{\pi}\left[\cos\alpha - \cos(\pi+\alpha)\right] \quad ; \qquad \cos(\pi+\alpha) = -\cos\alpha$$

$$V_{O(dc)} = V_{dc} = \frac{V_m}{\pi}\left[\cos\alpha + \cos\alpha\right]$$

$$V_{O(dc)} = V_{dc} = \frac{2V_m}{\pi}\cos\alpha$$

The above equation can be plotted to obtain the control characteristic of a single phase full wave controlled rectifier with RL load assuming continuous load current operation.

Normalizing the dc output voltage with respect to its maximum value, the normalized dc output voltage is given by

$$V_{dcn} = V_n = \frac{V_{dc}}{V_{dc(\max)}} = \frac{\frac{2V_m}{\pi}(\cos\alpha)}{\frac{2V_m}{\pi}} = \cos\alpha$$

$$V_{dcn} = V_n = \cos\alpha$$

Trigger angle α in degrees	$V_{O(dc)}$	Remarks
0	$V_{dm} = \left(\frac{2V_m}{\pi}\right)$	Maximum dc output voltage $V_{dc(max)} = V_{dm} = \left(\frac{2V_m}{\pi}\right)$
30^0	$0.866\ V_{dm}$	
60^0	$0.5\ V_{dm}$	
90^0	$0\ V_{dm}$	
120^0	$-0.5\ V_{dm}$	
150^0	$-0.866\ V_{dm}$	
180^0	$-V_{dm} = -\left(\frac{2V_m}{\pi}\right)$	

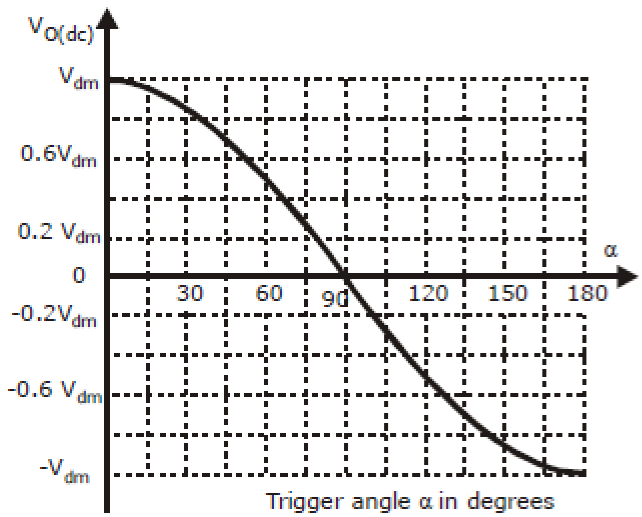


Fig.18 Control Characteristic

We notice from the control characteristic that by varying the trigger angle α we can vary the output dc voltage across the load. Thus it is possible to control the dc output voltage by changing the trigger angle α. For trigger angle α in the range of 0 to 90 degrees $\left(i.e.,\ 0 \le \alpha \le 90_0\right)$, V_{dc} is positive and the circuit operates as a controlled rectifier to convert ac supply voltage into dc output power which is fed to the load.

For trigger angle $\alpha > 90^0$, cos α becomes negative and as a result the average dc output voltage V_{dc}becomes negative, but the load current flows in the same positive direction. Hence the output power becomes negative. This means that the power flows from the load circuit to the input ac source. This is referred to as *linecommutated inverter operation*. During the inverter mode operation for $\alpha > 90^0$ the load energy can be fed back from the load circuit to the input ac source.

TO DERIVE AN EXPRESSION FOR RMS OUTPUT VOLTAGE

$$V_{O(RMS)} = \left[\frac{2}{2\pi} \int_{\alpha}^{(\pi+\alpha)} v_O^2 . d(\omega t) \right]^{\frac{1}{2}}$$

$$V_{O(RMS)} = \left[\frac{1}{\pi} \int_{\alpha}^{(\pi+\alpha)} V_m^2 \sin^2 \omega t . d(\omega t) \right]^{\frac{1}{2}}$$

$$V_{O(RMS)} = \left[\frac{V_m^2}{\pi} \int_{\alpha}^{(\pi+\alpha)} \sin^2 \omega t . d(\omega t) \right]^{\frac{1}{2}}$$

$$V_{O(RMS)} = \left[\frac{V_m^2}{\pi} \int_{\alpha}^{(\pi+\alpha)} \frac{(1-\cos 2\omega t)}{2} . d(\omega t) \right]^{\frac{1}{2}}$$

$$V_{O(RMS)} = V_m \left[\frac{1}{2\pi} \left\{ \int_{\alpha}^{(\pi+\alpha)} d(\omega t) - \int_{\alpha}^{(\pi+\alpha)} \cos 2\omega t . d(\omega t) \right\} \right]^{\frac{1}{2}}$$

$$V_{O(RMS)} = V_m \left[\frac{1}{2\pi} \left\{ (\omega t) \Big/_{\alpha}^{(\pi+\alpha)} - \left(\frac{\sin 2\omega t}{2} \right) \Big/_{\alpha}^{(\pi+\alpha)} \right\} \right]^{\frac{1}{2}}$$

$$V_{O(RMS)} = V_m \left[\frac{1}{2\pi} \left\{ (\pi + \alpha - \alpha) - \left(\frac{\sin 2(\pi+\alpha) - \sin 2\alpha}{2} \right) \right\} \right]^{\frac{1}{2}}$$

$$V_{O(RMS)} = V_m \left[\frac{1}{2\pi} \left\{ (\pi) - \left(\frac{\sin 2\pi \times \cos 2\alpha + \cos 2\pi \times \sin 2\alpha - \sin 2\alpha}{2} \right) \right\} \right]^{\frac{1}{2}}$$

$$V_{O(RMS)} = V_m \left[\frac{1}{2\pi} \left\{ (\pi) - \left(\frac{0 + \sin 2\alpha - \sin 2\alpha}{2} \right) \right\} \right]^{\frac{1}{2}}$$

$$V_{O(RMS)} = V_m \left[\frac{1}{2\pi} (\pi) \right]^{\frac{1}{2}} = \frac{V_m}{\sqrt{2}}$$

SINGLE PHASE SEMICONVERTERS

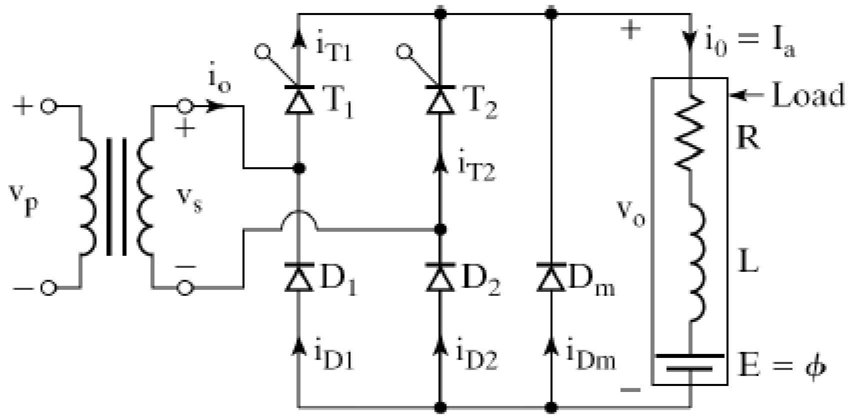


Fig.19 Circuit Diagram

Consider diode $_2D$ as $_1D$ in the figure and diode $_1D$ as $_2D$

Single phase semi-converter circuit is a full wave half controlled bridge converter which uses two thyristors and two diodes connected in the form of a full wave bridge configuration.
The two thyristors are controlled power switches which are turned on one after the other by applying suitable gating signals (gate trigger pulses). The two diodes are uncontrolled power switches which turn-on and conduct one after the other as and when they are forward biased.
The circuit diagram of a single phase semi-converter (half controlled bridge converter) is shown in the above figure with highly inductive load and a dc source in the load circuit. When the load inductance is large the load current flows continuously and we can consider the continuous load current operation assuming constant load current, with negligible current ripple (i.e., constant and ripple free load current operation).
The ac supply to the semiconverter is normally fed through a mains supply transformer having suitable turns ratio. The transformer is suitably designed to supply the required ac supply voltage (secondary output voltage) to the converter.
During the positive half cycle of input ac supply voltage, when the transformer secondary output line ‘A’ is positive with respect to the line ‘B’ the thyristor T_1 and the diode $D_1$are both forward biased. The thyristor $T_1$is triggered at $\omega t = \alpha$; $(0 \leq \alpha \leq \pi)$ by applying an appropriate gate trigger signal to the gate of T_1. The current in the circuit flows through the secondary line ‘A’,

through T_1, through the load in the downward direction, through diode D_1back to the secondary line 'B'.

$T_1$and D_1conduct together from $\omega t = \alpha$ to π and the load is connected to the input ac supply. The output load voltage follows the input supply voltage (the secondary output voltage of the transformer) during the period $\omega t = \alpha$ to π.

At $\omega t = \pi$, the input supply voltage decreases to zero and becomes negative during the period $\omega t = \pi$ to $(\pi + \alpha)$. The freewheeling diode $D_m$across the load becomes forward biased and conducts during the period $\omega t = \pi$ to $(\pi + \alpha)$.

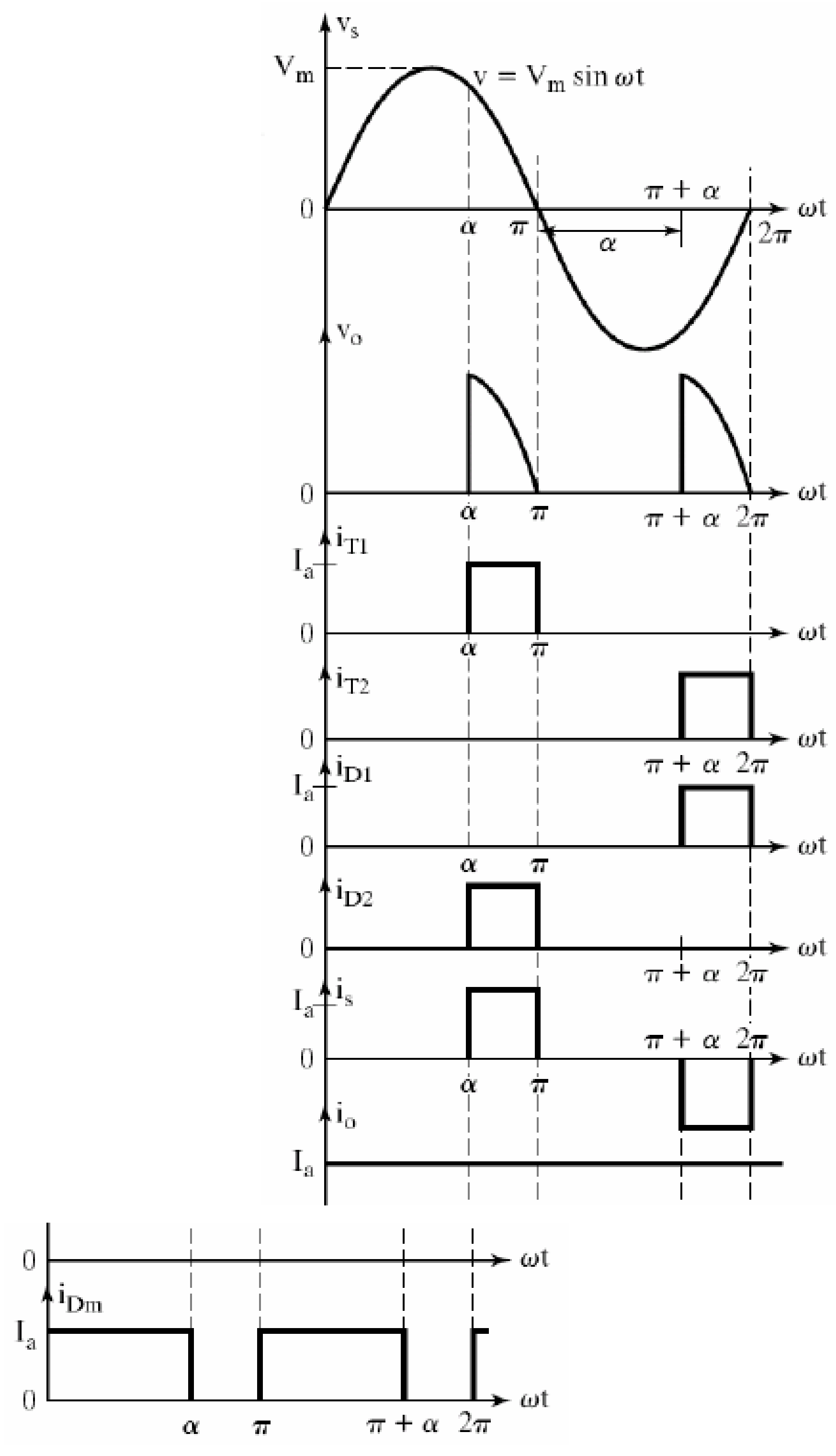


Fig.20 Waveforms of single phase semi-converter for RLE load and constant load current for $\alpha > 90^0$

The load current is transferred from $T_1$and $D_1$to the FWD D_m. $T_1$and $D_1$are turned off. The load current continues to flow through the FWD D_m. The loadcurrent free wheels (flows continuously) through the FWD during the freewheelingtime period π to $(\pi+\alpha)$.

During the negative half cycle of input supply voltage the secondary line 'A' becomes negative with respect to line 'B'. The thyristor $T_2$and the diode $D_2$are both forward biased. T_2 is triggered at $\omega t = (\pi+\alpha)$, during the negative half cycle. The FWD is reverse biased and turns-off as soon as T_2 is triggered. The load current continues to flow through T_2 and D_2 during the period $\omega t = (\pi+\alpha)$ to 2π.

TO DERIVE AN EXPRESSION FOR THE AVERAGE OR DC OUTPUT VOLTAGE OF A SINGLE PHASE SEMI-CONVERTER

The average output voltage can be found from

$$V_{dc} = \frac{2}{2\pi}\int_{\alpha}^{\pi} V_m \sin\omega t . d(\omega t)$$

$$V_{dc} = \frac{2V_m}{2\pi}\left[-\cos\omega t\right]_{\alpha}^{\pi}$$

$$V_{dc} = \frac{V_m}{\pi}\left[-\cos\pi + \cos\alpha\right] \;\; ; \; \cos\pi = -1$$

$$V_{dc} = \frac{V_m}{\pi}\left[1+\cos\alpha\right]$$

V_{dc} can be varied from $\frac{2V_m}{\pi}$ to 0 by varying α from 0 to π.

The maximum average output voltage is

$$\boldsymbol{V_{dc(max)} = V_{dm} = \frac{2V_m}{\pi}}$$

Normalizing the average output voltage with respect to its maximum value

$$V_{dcn} = V_n = \frac{V_{dc}}{V_{dm}} = 0.5(1+\cos\alpha)$$

The output control characteristic can be plotted by using the expression for V_{dc}

TO DERIVE AN EXPRESSION FOR THE RMS OUTPUT VOLTAGE OF A SINGLE PHASE SEMI-CONVERTER

The rms output voltage is found from

$$V_{O(RMS)} = \left[\frac{2}{2\pi}\int_{\alpha}^{\pi} V_m^2 \sin^2 \omega t . d(\omega t)\right]^{\frac{1}{2}}$$

$$V_{O(RMS)} = \left[\frac{V_m^2}{2\pi}\int_{\alpha}^{\pi} (1-\cos 2\omega t) . d(\omega t)\right]^{\frac{1}{2}}$$

$$V_{O(RMS)} = \frac{V_m}{\sqrt{2}}\left[\frac{1}{\pi}\left(\pi - \alpha + \frac{\sin 2\alpha}{2}\right)\right]^{\frac{1}{2}}$$

SINGLE PHASE FULL CONVERTER (FULLY CONTROLLED BRIDGE CONVERTER)

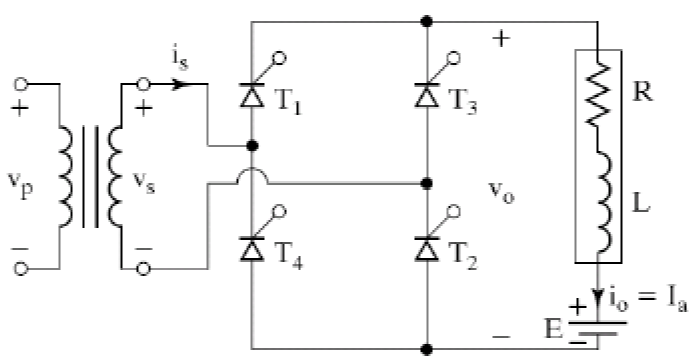


Fig. 21 Circuit Diagram

The circuit diagram of a single phase fully controlled bridge converter is shown in the figure with a highly inductive load and a dc source in the load circuit so that the load current is continuous and ripple free (constant load current operation).

The fully controlled bridge converter consists of four thyristors T_1, T_2, T_3 and T_4 connected in the form of full wave bridge configuration as shown in the figure. Each thyristor is controlled and turned on by its gating signal and naturally turns off when a reverse voltage appears across it.

During the positive half cycle when the upper line of the transformer secondary winding is at a positive potential with respect to the lower end the thyristors $T_1$and $T_2$are forward biased during the time interval $\omega t = 0$ to π. The thyristors $T_1$and $T_2$are triggered simultaneously $\omega t = \alpha$; $(0 \leq \alpha \leq \pi)$, the load is connected to the input supply through the conducting thyristors $T_1$and T_2. The output voltage across the load follows the input supply voltage and hence output voltage $V_O = V_m \sin \omega t$. Due to the inductive load T_1 and T_2will continue to conduct beyond $\omega t = \pi$, even though the input voltage becomes negative. T_1 and T_2 conduct together during the time period α to $(\pi + \alpha)$, for a time duration of π radians (conduction angle of each thyristor = 180^0).

During the negative half cycle of input supply voltage for $\omega t = \pi$ to 2π the thyristors $T_3$and $T_4$are forward biased. T_3 and $T_4$are triggered at $\omega t = (\pi + \alpha)$. As soon as the thyristors T_3 and T_4 are triggered a reverse voltage appears across the thyristors $T_1$and $T_2$and they naturally turn-off and the load current is transferred from $T_1$and $T_2$to the thyristors $T_3$and T_4. The output voltage across the load follows the supply voltage and $V_O = -V_m \sin \omega t$ during the time period $\omega t = (\pi + \alpha)$ to $(2\pi + \alpha)$. In the next positive half cycle when $T_1$and T_2 are triggered, $T_3$and $T_4$are reverse biased and they turn-off. The figure shows the waveforms of the input supply voltage, the output load voltage, the constant load current with negligible ripple and the input supply current.

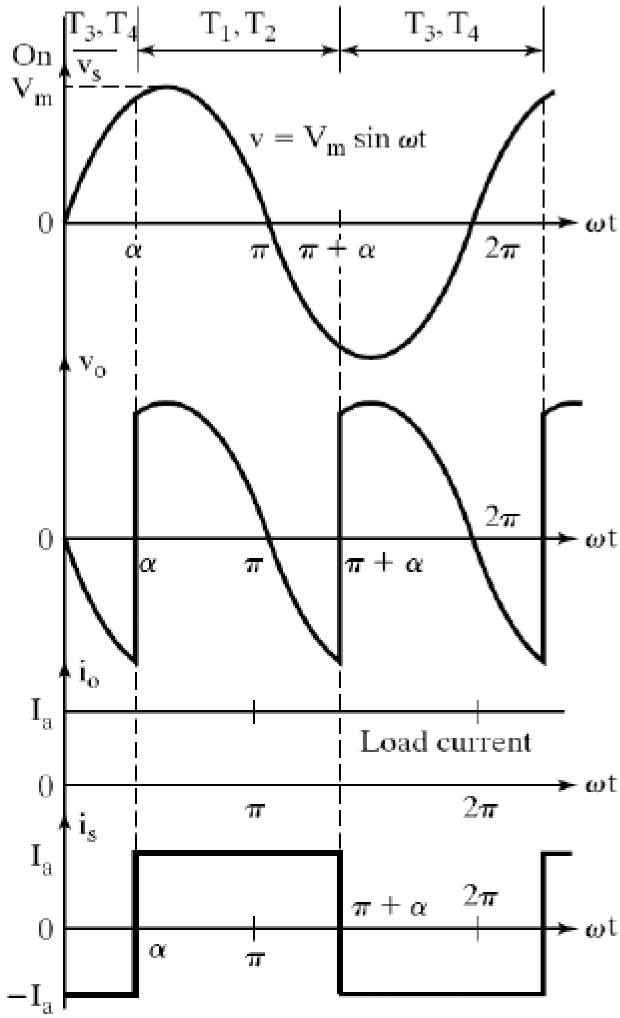


Fig.22 Wave Forms

During the time period $\omega t = \alpha$ to π, the input supply voltage V_S and the input supply current i_s are both positive and the power flows from the supply to the load. The converter operates in the rectification mode during $\omega t = \alpha$ to π.

During the time period $\omega t = \pi$ to $(\pi + \alpha)$, the input supply voltage v_s is negative and the input supply current i_s is positive and there will be reverse power flow from the load circuit to the input supply. The converter operates in the inversion mode during the time period $\omega t = \pi$ to $(\pi + \alpha)$ and the load energy is fed back to the input source.

The single phase full converter is extensively used in industrial applications up to about 15kW of output power. Depending on the value of trigger angle α, the average output voltage may be either positive or negative and two quadrant operation is possible.

TO DERIVE AN EXPRESSION FOR THE AVERAGE (DC) OUTPUT VOLTAGE

The average (dc) output voltage can be determined by using the expression

$$V_{O(dc)} = V_{dc} = \frac{1}{2\pi}\left[\int_0^{2\pi} v_O.d(\omega t)\right] \ ;$$

The output voltage waveform consists of two output pulses during the input supply time period between 0 & 2 π radians . In the continuous load current operation of a single phase full converter (assuming constant load current) each thyristor conduct for π radians (180^0) after it is triggered. When thyristors $T_1$and $T_2$are triggered at $\omega t = \alpha$ $_1T$ and $_2T$ conduct from α to $(\pi + \alpha)$ and the output voltage follows the input supply voltage. Therefore output voltage $V_O = V_m \sin \omega t$ for $\omega t = \alpha$ to $(\pi + \alpha)$.

Hence the average or dc output voltage can be calculated as

$$V_{O(dc)} = V_{dc} = \frac{2}{2\pi}\left[\int_\alpha^{\pi+\alpha} V_m \sin \omega t.d(\omega t)\right]$$

$$V_{O(dc)} = V_{dc} = \frac{1}{\pi}\left[\int_\alpha^{\pi+\alpha} V_m \sin \omega t.d(\omega t)\right]$$

$$V_{O(dc)} = V_{dc} = \frac{V_m}{\pi}\left[\int_\alpha^{\pi+\alpha} \sin \omega t.d(\omega t)\right]$$

$$V_{O(dc)} = V_{dc} = \frac{V_m}{\pi}\left[-\cos \omega t\right]_\alpha^{\pi+\alpha}$$

$$V_{O(dc)} = V_{dc} = \frac{V_m}{\pi}\left[-\cos(\pi+\alpha)+\cos\alpha\right] \quad ;$$

$$\cos(\pi+\alpha) = -\cos\alpha$$

Therefore $V_{O(dc)} = V_{dc} = \frac{2V_m}{\pi}\cos\alpha$

The dc output voltage V_{dc} can be varied from a maximum value of $\frac{2V_m}{\pi}$ for

$\alpha = 0^0$ *to a minimum value of* $\frac{-2V_m}{\pi}$ *for* $\alpha = \pi$ *radians*

The maximum average dc output voltage is calculated for a trigger angle $\alpha = 0^0$ and is obtained as

$$V_{dc(max)} = V_{dm} = \frac{2V_m}{\pi}\times\cos(0) = \frac{2V_m}{\pi}$$

Therefore $\quad V_{dc(max)} = V_{dm} = \frac{2V_m}{\pi}$

The normalized average output voltage is given by

$$V_{dcn} = V_n = \frac{V_{O(dc)}}{V_{dc(max)}} = \frac{V_{dc}}{V_{dm}}$$

$$V_{dcn} = V_n = \frac{\frac{2V_m}{\pi}\cos\alpha}{\frac{2V_m}{\pi}} = \cos\alpha$$

Therefore $V_{dcn} = V_n = \cos\alpha$; for a single phase full converter assuming continuous and constant load current operation.

CONTROL CHARACTERISTIC OF SINGLE PHASE FULL CONVERTER

The dc output control characteristic can be obtained by plotting the average or dc output voltage V_{dc} versus the trigger angle α

For a single phase full converter the average dc output voltage is given by the equation

$$V_{O(dc)} = V_{dc} = \frac{2V_m}{\pi}\cos\alpha$$

Trigger angle α in degrees	$V_{O(dc)}$	Remarks
0	$V_{dm} = \left(\frac{2V_m}{\pi}\right)$	Maximum dc output voltage $V_{dc(\max)} = V_{dm} = \left(\frac{2V_m}{\pi}\right)$
30^0	0.866 V_{dm}	
60^0	0.5 V_{dm}	
90^0	0 V_{dm}	
120^0	-0.5 V_{dm}	
150^0	-0.866 V_{dm}	
180^0	$-V_{dm} = -\left(\frac{2V_m}{\pi}\right)$	

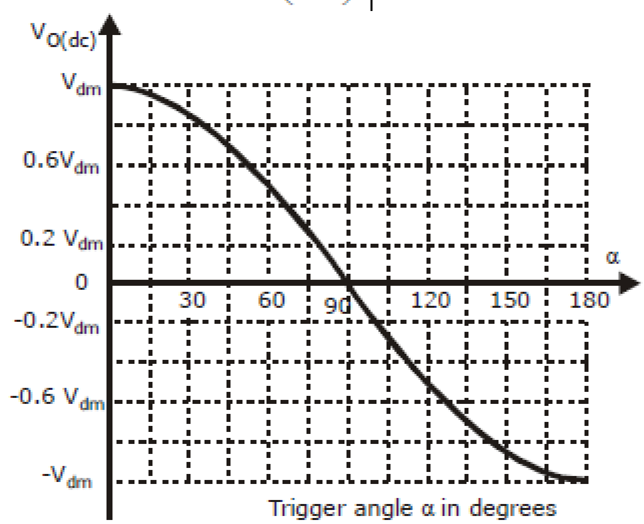
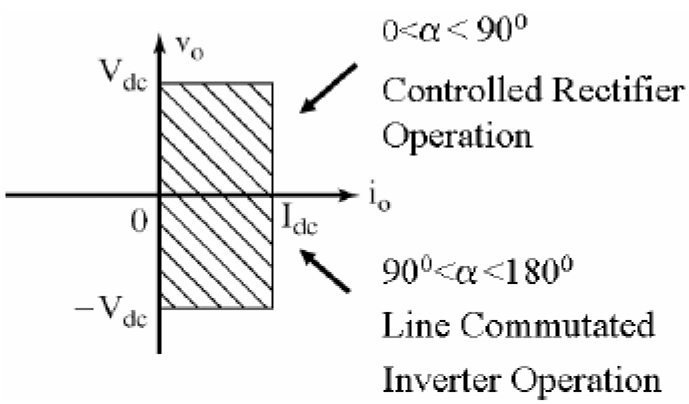


Fig.23 Control Characteristic

We notice from the control characteristic that by varying the trigger angle α we can vary the output dc voltage across the load. Thus it is possible to control the dc output voltage by changing the trigger angle α. For trigger angle α in the range of 0 to 90 degrees $\left(i.e., 0 \leq \alpha \leq 90^0\right)$, $V_{dc}$is positive and the average dc load curren $I_{dc}$is also positive. The average or dc output power $P_{dc}$is positive, hence the circuit operates as a controlled rectifier to convert ac supply voltage into dc output power which is fed to the load.

For trigger angle $\alpha > 90^0$, $\cos\alpha$ becomes negative and as a result the average dc output voltage V_{dc} becomes negative, but the load current flows in the same positive direction i.e., I_{dc} is positive . Hence the output power becomes negative. This means that the power flows from the load circuit to the input ac source. This is referred to as *line commutated inverter operation*. During the inverter mode operation for $\alpha > 90^0$ the load energy can be fed back from the load circuit to the input ac source

TWO QUADRANT OPERATION OF A SINGLE PHASE FULL CONVERTER



The above figure shows the two regions of single phase full converter operation in the V_{dc} versus I_{dc} plane. In the first quadrant when the trigger angle α is less than 90^0, and V_{dc} I_{dc} are both positive and the converter operates as a controlled rectifier and converts the ac input power into dc output power. The power flows from the input source to the load circuit. This is the normal controlled rectifier operation where P_{dc} is positive.

When the trigger angle is increased above 90^0, V_{dc} becomes negative but I_{dc} is positive and the average output power (dc output power) P_{dc} becomes negative andthe power flows from the load circuit to the input source. The operation occurs in thefourth quadrant where V_{dc} is negative and I_{dc} is positive. The converter operates as aline commutated inverter.

TO DERIVE AN EXPRESSION FOR THE RMS VALUE OF THE OUTPUT VOLTAGE

The rms value of the output voltage is calculated as

$$V_{O(RMS)} = \sqrt{\frac{1}{2\pi}\left[\int_0^{2\pi} v_O^2 . d(\omega t)\right]}$$

The single phase full converter gives two output voltage pulses during the input supply time period and hence the single phase full converter is referred to as a two pulse converter. The rms output voltage can be calculated as

$$V_{O(RMS)} = \sqrt{\frac{2}{2\pi}\left[\int_\alpha^{\pi+\alpha} v_O^2 . d(\omega t)\right]}$$

$$V_{O(RMS)} = \sqrt{\frac{1}{\pi}\left[\int_\alpha^{\pi+\alpha} V_m^2 \sin^2 \omega t . d(\omega t)\right]}$$

$$V_{O(RMS)} = \sqrt{\frac{V_m^2}{\pi}\left[\int_\alpha^{\pi+\alpha} \sin^2 \omega t . d(\omega t)\right]}$$

$$V_{O(RMS)} = \sqrt{\frac{V_m^2}{\pi}\left[\int_\alpha^{\pi+\alpha} \frac{(1-\cos 2\omega t)}{2} . d(\omega t)\right]}$$

$$V_{O(RMS)} = \sqrt{\frac{V_m^2}{2\pi}\left[\int_\alpha^{\pi+\alpha} d(\omega t) - \int_\alpha^{\pi+\alpha} \cos 2\omega t . d(\omega t)\right]}$$

$$V_{O(RMS)} = \sqrt{\frac{V_m^2}{2\pi}\left[(\omega t)\Big/_{\alpha}^{\pi+\alpha} - \left(\frac{\sin 2\omega t}{2}\right)\Big/_{\alpha}^{\pi+\alpha}\right]}$$

$$V_{O(RMS)} = \sqrt{\frac{V_m^2}{2\pi}\left[(\pi+\alpha-\alpha) - \left(\frac{\sin 2(\pi+\alpha) - \sin 2\alpha}{2}\right)\right]}$$

$$V_{O(RMS)} = \sqrt{\frac{V_m^2}{2\pi}\left[(\pi) - \left(\frac{\sin(2\pi+2\alpha) - \sin 2\alpha}{2}\right)\right]} \quad ; \quad \sin(2\pi+2\alpha) = \sin 2\alpha$$

$$V_{O(RMS)} = \sqrt{\frac{V_m^2}{2\pi}\left[(\pi)-\left(\frac{\sin 2\alpha - \sin 2\alpha}{2}\right)\right]}$$

$$V_{O(RMS)} = \sqrt{\frac{V_m^2}{2\pi}(\pi)-0} = \sqrt{\frac{V_m^2}{2}} = \frac{V_m}{\sqrt{2}}$$

$$V_{O(RMS)} = \frac{V_m}{\sqrt{2}} = V_S$$

Hence the rms output voltage is same as the rms input supply voltage

The rms thyristor current can be calculated as

Each thyristor conducts for π radians or 180^0in a single phase full converter operating at continuous and constant load current.

Therefore rms value of the thyristor current is calculated as

$$I_{T(RMS)} = I_{O(RMS)}\sqrt{\frac{\pi}{2\pi}} = I_{O(RMS)}\sqrt{\frac{1}{2}}$$

$$I_{T(RMS)} = \frac{I_{O(RMS)}}{\sqrt{2}}$$

The average thyristor current can be calculated as

$$I_{T(Avg)} = I_{O(dc)} \times \left(\frac{\pi}{2\pi}\right) = I_{O(dc)} \times \left(\frac{1}{2}\right)$$

$$I_{T(Avg)} = \frac{I_{O(dc)}}{2}$$

SINGLE PHASE DUAL CONVERTER

We have seen in the case of a single phase full converter with inductive loads the converter can operate in two different quadrants in the versus $V_{dc}I_{dc}$operating diagram. If two single phase full converters are connected in parallel and in opposite direction (connected in back to back) across a common load four quadrant operation is possible. Such a converter is called as a dual converter which is shown in the figure.

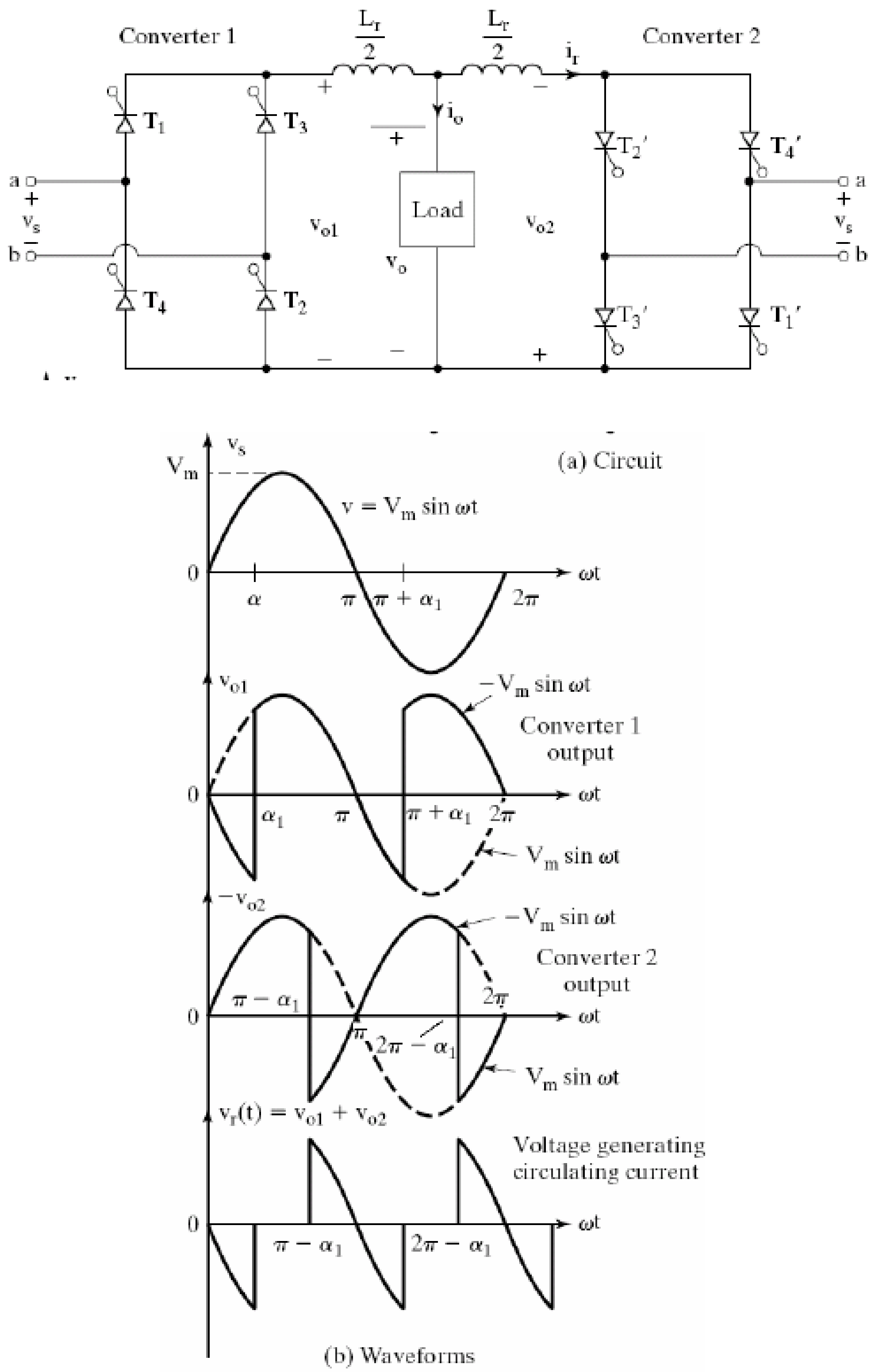


Fig.24 Circuit Diagram & Wave Forms

The dual converter system will provide four quadrant operations and is normally used in high power industrial variable speed drives. The converter number 1 provides a positive dc output voltage and a positive dc load current, when operated in the rectification mode.

The converter number 2 provides a negative dc output voltage and a negative dc load current when operated in the rectification mode. We can thus have bidirectional load current and bi-directional dc output voltage. The magnitude of output dc load voltage and the dc load current can be controlled by varying the trigger angles α_1 & α_2 of the converters 1 and 2 respectively.

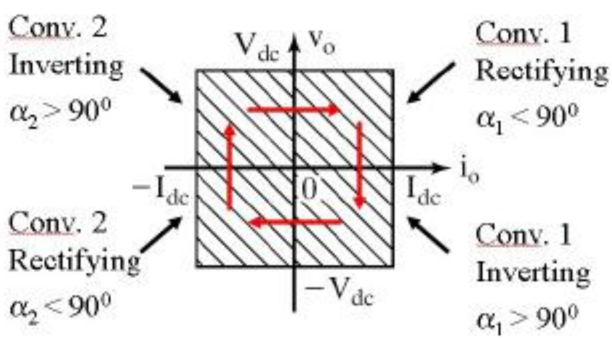


Fig.25 Four quadrant operation of a dual converter

There are two modes of operations possible for a dual converter system.

- Non circulating current mode of operation (circulating current free mode of operation).
- Circulating current mode of operation.

NON CIRCULATING CURRENT MODE OF OPERATION (CIRCULATING CURRENT FREE MODE OF OPERATION)

In this mode of operation only one converter is switched on at a time while the second converter is switched off. When the converter 1 is switched on and the gate trigger signals are released to the gates of thyristors in converter 1, we get an average output voltage across the load, which can be varied by adjusting the trigger angle α_1 of the converter 1. If α_1 is less than 90^0, the converter 1 operates as a controlled rectifier and converts the input ac power into dc output power to feed the load. V_{dc} and $I_{dc}$are both positive and the operation occurs in the first quadrant. The averageoutput power $P_{dc}=V_{dc}\times I_{dc}$ is positive. The power flows from the input ac supply tothe load. When $\alpha_1$is increased above 90^0converter 1 operates as a line commutated inverter and V_{dc}becomes negative while $I_{dc}$is positive and the output power P_{dc}becomes negative. The power is fed back from the load circuit to the input ac sourcethrough the converter 1. The load current falls to zero when the load energy is utilizedcompletely.

The second converter 2 is switched on after a small delay of about 10 to 20 mill seconds to allow all the thyristors of converter 1 to turn off completely. The gate signals are released to the thyristor gates of converter 2 and the trigger angle $\alpha_2$is adjusted such that $0^0\leq \alpha_2 \leq 90^0$ so that converter 2 operates as a controlled rectifier. The dc output voltage $V_{dc}$and $I_{dc}$are both negative and the load current flows in the reverse direction. The magnitude of $V_{dc}$and $I_{dc}$are controlled by the trigger angle α_2. The operation occurs in the third quadrant where $V_{dc}$and I_{dc} are both negative and output power $P_{dc}$is positive and the converter 2 operates as a controlled rectifier and converts the ac supply power into dc output power which is fed to the load.

When we want to reverse the load current flow so that $I_{dc}$is positive we have to operate converter 2 in the inverter mode by increasing the trigger angle $\alpha_2$above 90^0. When $\alpha_2$is made greater than 90^0, the converter 2 operates as a line commutated inverter and the load power (load energy) is fed back to ac mains. The current falls to zero when all the load energy is utilized and the converter 1 can be switched on after a short delay of 10 to 20 milli seconds to ensure that the converter 2 thyristors are completely turned off.

The advantage of non circulating current mode of operation is that there is no circulating current flowing between the two converters as only one converter operates and conducts at a time while the other converter is switched off. Hence there is no need of the series current limiting inductors between the outputs of the two converters. The current rating of thyristors is low in this mode.

But the disadvantage is that the load current tends to become discontinuous and the transfer characteristic becomes non linear. The control circuit becomes complex and the output response is sluggish as the load current reversal takes some time due to the time delay between the switching off of one converter and the switching on of the other converter. Hence the output dynamic response is poor. Whenever a fast and frequent reversal of the load current is required, the dual converter is operated in the circulating current mode.

CIRCULATING CURRENT MODE OF OPERATION

In this mode of operation both the converters 1 and 2 are switched on and operated simultaneously and both the converters are in a state of conduction. If converter 1 is operated as a controlled rectifier by adjusting the trigger angle α_1 between 0 to 90^0 the second converter 2 is operated as a line commutated inverter byincreasing its trigger angle α_2 above 90^0. The trigger angles α_1 and α_2 are adjustedsuch that they produce the same average dc output voltage across the load terminals.

The average dc output voltage of converter 1 is

$$V_{dc1} = \frac{2V_m}{\pi}\cos\alpha_1$$

The average dc output voltage of converter 2 is

$$V_{dc2} = \frac{2V_m}{\pi}\cos\alpha_2$$

In the dual converter operation one converter is operated as a controlled rectifier with $\alpha_1 < 90$ and the second converter is operated as a line commutate inverter in the inversion mode with $\alpha_2 > 90$.

$$V_{dc1} = -V_{dc2}$$

$$\frac{2V_m}{\pi}\cos\alpha_1 = \frac{-2V_m}{\pi}\cos\alpha_2 = \frac{2V_m}{\pi}(-\cos\alpha_2)$$

$$\cos\alpha_1 = -\cos\alpha_2 \text{ or } \cos\alpha_2 = -\cos\alpha_1 = \cos(\pi - \alpha_1)$$

$$\alpha_2 = (\pi - \alpha_1) \quad \text{or} \quad (\alpha_1 + \alpha_2) = \pi \text{ radians}$$

$$\alpha_2 = (\pi - \alpha_1)$$

When the trigger angle $\alpha_1$of converter 1 is set to some value the trigger angle $\alpha_2$of the second converter is adjusted such that $\alpha_2 = (180^0 - \alpha_1)$. Hence forcirculating current mode of operation where both converters are conducting at thesame time $(\alpha_1 + \alpha_2) = 180^0$ so that they produce the same dc output voltage across the load.

When α_1<90 (say $\alpha_1 = 30^0$) the converter 1 operates as a controlled rectifierand converts the ac supply into dc output power and the average load current I_{dc}ispositive. At the same time the converter 2 is switched on and operated as a linecommutated inverter, by adjusting the trigger angle $\alpha_2$such that, $\alpha_2 = (180^0 - \alpha_1)$which is equal to 150^0, when $\alpha_1 = 30^0$. The converter 2 will operate in the inversionmode and feeds the load energy back to the ac supply. When we want to reverse theload current flow we have to switch the roles of the two converters.

When converter 2 is operated as a controlled rectifier by adjusting the trigger angle $\alpha_2$such that $\alpha_2 < 90^0$, the first converter1 is operated as a line commutated inverter, by adjusting the trigger angle $\alpha_1$such that $\alpha_1 > 90^0$. The trigger angle $\alpha_1$is adjusted such that $\alpha_1 = (180^0 - \alpha_2)$ for a set value of α_2.

In the circulating current mode a current builds up between the two converters even when the load current falls to zero. In order to limit the circulating current flowing between the two converters, we have to include current limiting reactors in series between the output terminals of the two converters.

The advantage of the circulating current mode of operation is that we can have faster reversal of load current as the two converters are in a state of conduction simultaneously. This greatly improves the dynamic response of the output giving a faster dynamic response. The output voltage and the load current can be linearly varied by adjusting the trigger angles α_1& $\alpha_2$to obtain a smooth and linear output control. The control circuit becomes relatively simple. The transfer characteristic between the output voltage and the trigger angle is linear and hence the output response is very fast. The load current is free to flow in either direction at any time. The reversal of the load current can be done in a faster and smoother way.

The disadvantage of the circulating current mode of operation is that a current flows continuously in the dual converter circuit even at times when the load current is zero. Hence we should connect current limiting inductors (reactors) in order to limit the peak circulating current within specified value. The circulating current flowing through the series inductors gives rise to increased power losses, due to dc voltage drop across the series inductors which decreases the efficiency. Also the power factor of operation is low. The current limiting series inductors are heavier and bulkier which increases the cost and weight of the dual converter system.

The current flowing through the converter thyristors is much greater than the dc load current. Hence the thyristors should be rated for a peak thyristor current of $I_{T(max)} = I_{dc(max)} + i_{r(max)}$ $, where\ I_{dc(max)}$ is the maximum dc load current and $i_{r(max)}$ is the maximum value of the circulating current.

TO CALCULATE THE CIRCULATING CURRENT

As the instantaneous output voltages of the two converters are out of phase, there will be an instantaneous voltage difference and this will result in circulating current between the two converters. In order to limit the circulating current, current limiting reactors are connected in series between the outputs of the two converters. This circulating current will not flow through the load and is normally limited by the current reactor L_r .

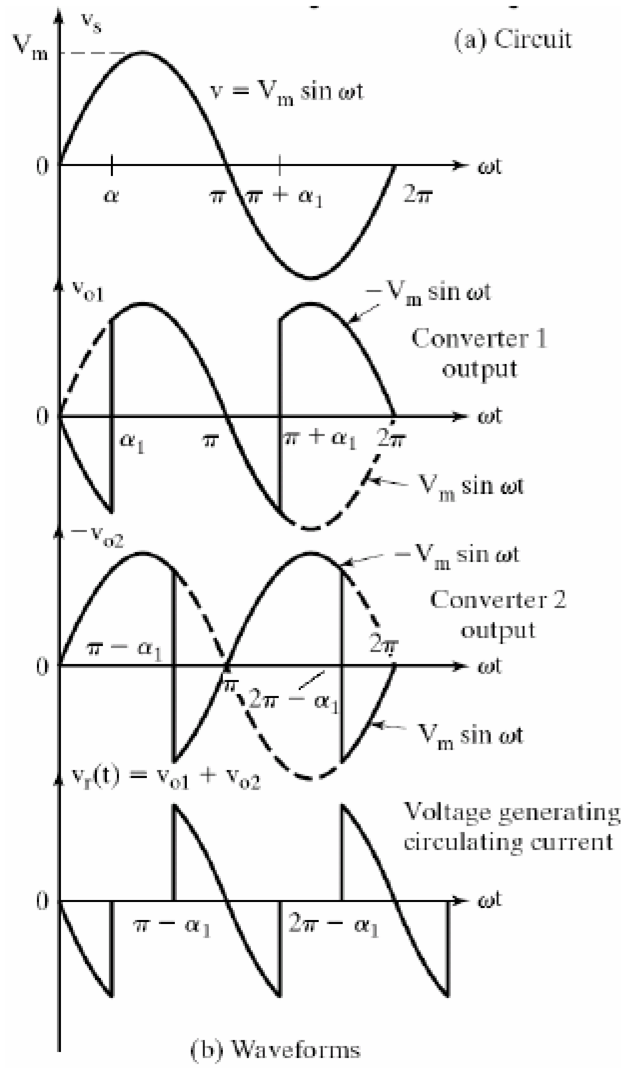


Fig.26 Waveforms of dual converter

If v_{O1} and v_{O2} are the instantaneous output voltages of the converters 1 and 2, respectively the circulating current can be determined by integrating the instantaneous voltage difference (which is the voltage drop across the circulating current reactor L_r), starting from $\omega t = (2\pi - \alpha_1)$. As the two average output voltages during the interval $\omega t = (\pi + \alpha_1)$ to $(2\pi - \alpha_1)$ are equal and opposite their contribution to the instantaneous circulating current i_r is zero.

$$i_r = \frac{1}{\omega L_r}\left[\int_{(2\pi-\alpha_1)}^{\omega t} v_r . d(\omega t)\right]; \quad v_r = (v_{O1} - v_{O2})$$

As the output voltage v_{O2} is negative

$$v_r = (v_{O1} + v_{O2})$$

Therefore $$i_r = \frac{1}{\omega L_r}\left[\int_{(2\pi-\alpha_1)}^{\omega t} (v_{O1} + v_{O2}) . d(\omega t)\right];$$

$$v_{O1} = -V_m \sin \omega t \text{ for } (2\pi - \alpha_1) \text{ to } \omega t$$

$$i_r = \frac{V_m}{\omega L_r}\left[\int_{(2\pi-\alpha_1)}^{\omega t} -\sin \omega t . d(\omega t) - \int_{(2\pi-\alpha_1)}^{\omega t} \sin \omega t . d(\omega t)\right]$$

$$i_r = \frac{V_m}{\omega L_r}\left[(\cos \omega t)\Big/_{(2\pi-\alpha_1)}^{\omega t} + (\cos \omega t)\Big/_{(2\pi-\alpha_1)}^{\omega t}\right]$$

$$i_r = \frac{V_m}{\omega L_r}\left[(\cos \omega t) - \cos(2\pi - \alpha_1) + (\cos \omega t) - \cos(2\pi - \alpha_1)\right]$$

$$i_r = \frac{V_m}{\omega L_r}\left[2\cos \omega t - 2\cos(2\pi - \alpha_1)\right]$$

$$i_r = \frac{2V_m}{\omega L_r}(\cos \omega t - \cos \alpha_1)$$

The instantaneous value of the circulating current depends on the delay angle.